

North Dakota Coordinate Reference System

Handbook and User Guide

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Definition of Terms

Affine transformation – Linear mapping method that preserves points, straight lines and planes.

Conformal map projection – A projection where the linear distortion is unique (the same in every direction) at a point. The versions of this projection type used for SPCS are the Lambert Conformal Conic, Transverse Mercator, and Hotine Oblique Mercator.

Coordinate System – A reference system for a unique location of a point in space.

Datum – A basis for measurements over a limited area of the earth with its origin at a location on the Earth's surface. A reference surface that serves to provide known locations to begin surveys and create maps.

Geoid – An approximation of the earth's surface that is approximately equal to mean sea level extended over and under land areas.

Linear distortion – For conformal map projections, it is the amount by which a distance or length in a projected coordinate reference system from the actual distance on or near the topographical surface of the Earth. Also know as "scale error" when evaluated with respect to distances on the reference ellipsoid surface.

Low Distortion Projection – A map projection that minimizes linear distortion.

Map Projection – The representation of the earth's sphere onto a plane surface.

Raster – A grid of individual pixels that compose an image.

Reference Ellipsoid – An oblate ellipsoid of revolution that approximates the size and shape of the entire Earth geoid (mean sea level) or a large portion of it. When oriented with respect to a geometric reference frame or datum, it defines the reference surface for projected coordinate reference systems.

State plane grid – Plane coordinate system with perpendicular north-south and east-west lines

Topographic surface – The arrangement of physical features on land surfaces.

Vector – Comprised of paths defined by a start and end point on a Cartesian plane.

Zone centroid – Projection of a region of the Earth's surface to sea level at geographical center.

Abbreviations

3DEP – USGS 3D Elevation Program

CAD – Computer Aided Design

DEM - Digital Elevation Model

EPSG – European Petroleum Survey Group

GRS 80 – Reference Ellipsoid

GIS – Geographic Information System

GML – Geographic Markup Language

GPS – Global Positioning System

ift – International foot

LCC – Lambert Conformal Conic

LDP – Low Distortion Projection

m – meter

NAD 27 – North American Datum of 1927

NAD 83 – North American Datum of 1983

NATRF 2022 – North American Terrestrial Reference Frame of 2022

NAVD 88 - North American Vertical Datum of 1988

NDCRS – North Dakota Coordinate Reference System

NDDOT – North Dakota Department of Transportation

NGS – National Geodetic Survey

NOAA – National Oceanic and Atmospheric Administration

OM – Oblique Mercator

ppm – parts per million

Prj – Esri projection files

SPCS 1983 – State Plane Coordinate System 1983

SPCS 2022 – State Plane Coordinate System 2022

TM – Transverse Mercator

TRF – Terrestrial Reference Frame

WKT – Well Know Text

Introduction

Project Background

In April 2018 the Department of Commerce advertised action of notice to a proposed change to the State Plane Coordinate system. Under direction from the Department of Commerce, National Oceanic Atmospheric Administration's National Geodetic Survey will establish the State Plane Coordinate System of 2022 which supersedes previous coordinate systems. NGS allowed stakeholders to create a Low Distortion Projection coordinate system to be included as part of the SPCS 2022.

The purpose of this project is to design a multi-zone LDP coordinate system to provide coverage for North Dakota. Currently the North Dakota Department of Transportation uses State Plane Coordinate system of 83 as the basis for all state and federal highway projects. North Dakota has 53 counties spanning across an area approximately 350 miles east to west and 210 miles north to south. To achieve lower ground distortion state plane grid coordinates are projected to a ground coordinate using a combination factor created for each county.

The objective of the LDP coordinate system is to provide coverage over the entire state with as few zones as possible to achieve ± 20 ppm (1:50,000) distortion for 95% of the state. Analysis and design of the LDPs was performed with software and digital topographic height models with design alternatives limited to Transverse Mercator and Lambert Conformal Conic projections. Design parameters were defined such that all zones of the final selected coordinate system are compatible with a wide range of commonly used commercial surveying, engineering, and GIS software.

NGS will create one statewide zone designed to a minimum of ± 50 ppm distortion for SPCS 2022. To meet current and future surveying and design needs, having an LDP coordinate system referenced to the topographic surface will reduce projection distortion for surveying and mapping projects and eliminate the need to modify the SPCS with a combination factor. Together with the ND Society of Professional Land Surveyors, ND Department of Water Resources and ND State Geospatial Committee the NDDOT led the effort to design a multi-zone LDP layer to be a part of North Dakota's SPCS 2022.

Project Justification

The current NDDOT method to create project coordinates is to scale SPCS 83 grid coordinates to projected ground coordinates on the topographical surface. This approach reduces linear distortion at ground level but it does not optimally minimize distortion for projects with a large extent. NGS published their policies and procedures for the development of the SPCS 2022 including a provision to allow states to design zones with linear distortion below the ± 50 ppm (1:20,000) criterion that NGS uses for SPCS 2022. The NDDOT opted to design a new LDP coordinate system for inclusion in SPCS 2022. Comprised of 16 LDP zones the new system has the following advantages over the previous scaled approach with SPCS 83:

- Optimally minimizes linear distortion over the largest areas possible so that ground coordinates can be obtained without additional scaling or modification.
- Simplifies data management by reducing the number of coordinate systems to 16 LDP zones.

- Facilitates data transferability and management with LDPs that are based on rigorously defined projected coordinate systems supported across a broad range of geospatial platforms.
- Simplifies surveying workflows by eliminating the need to perform vendor-specific methods of scaling, rotating, and translating spatial data including horizontal calibrations or localization.
- Generates coordinates that are distinct from previous coordinate systems and zone-specific coordinates that are easily identifiable.

Project Deliverables

Deliverables for this project consist of:

- Distortion maps: 11" x 17" distortion maps for each LDP zones and one map of the entire state showing all LDP zones. 8.5" x 11" index map of the state. These maps are available in PDF and PNG format as well as included as figures in this report.
- ArcMap V10.7 map packages files corresponding to all maps.
- GIS raster datasets of linear distortion, topographic height, geoid height, and hillshade, in IMG format and referenced to NAD 83 (2011) epoch 2010.00. The linear distortion rasters are in units of parts per million. The NDCRS distortion rasters are at 3 arc-second resolution for each zone and the entire state. Metadata (ISO 19139 compliant) also include for each raster.
- GIS vector feature dataset corresponding to the ArcMap documents and consisting of Esri shapefiles, referenced to NAD 83 (2011) epoch 2010.00. Includes NDCRS distortion contours, zone polygons projection axes, and zone centroid points. Metadata (ISO 19139 compliant) also included for each feature class.
- Esri projection (*.prj) files for each of the 16 zones of the final NDCRS definitions.
- Microsoft PowerPoint presentation to be used for training sessions. The presentation includes the project background, details about map projections, distortion maps, performance statistics, and zone parameters.
- Microsoft Excel spreadsheets of NDCRS projection parameters and centroid coordinates.

The North Dakota Coordinate Reference System

Design Criteria and Parameters

The following criteria were used to design the North Dakota Coordinate Reference System:

1. **Design Criteria:** The target accuracy was for linear distortion to be within ± 20 ppm (± 0.1 ft per mile or 1:50,000) for 95% of the state. Linear distortion is the difference in distance between a pair of projected grid coordinates as compared to the actual horizontal distance on the topographical ground surface of the Earth.
2. **Projection Types:** The 3 commonly used conformal projection types are allowed by NGS for inclusion in the SPCS 2022 include Lambert Conformal Conic, Transverse Mercator, and Oblique Mercator. The NDCRS designs were limited to LCC and TM projections. It should be noted that conformal projections are ideal because linear distortion is the same in every direction from a point. Review NGS procedures document NGS 2023-1214-03-A3 for technical specifications for SPCS 2022 designs.
3. **Geodetic Reference System:** NAD 83 is the *defining* datum for the NDCRS. SPCS 2022 will use the NATRF 2022 datum. Both datums use the Geodetic Reference System of 1980 as the reference ellipsoid.
4. **Zone Configuration:** The NDCRS zones were created to minimize the number of zones while achieving NGS's design criteria for linear distortion with the following criteria:
 - a. Each zone consists of aggregated counties bounded by county lines.
 - b. The total number of zones is 16, compiled from 53 counties with no zone overlap.
 - c. Zones are numbered from left to right, top to bottom.
 - d. Zones are named after the largest populated city in the zone based on recent Census information.
5. **Uniqueness:** NGS policy for SPCS 2022 zones specifies each zone must have a unique combination of origin latitude, longitude, and projection axis that is not repeated. The policy also specifies that all zones must differ by at least 10,000 meters horizontally from previous versions of SPCS, UTM, and other SPCS 2022 zone layers covering the same geographic region. Following NGS policy NDCRS was designed so the number values differ substantially from those of the following coordinate systems:
 - a. State Plane Coordinate System of 2022, North Dakota Zone
 - b. State Plane Coordinate System of 1983, North Dakota North and South zones, meters
 - c. State Plane Coordinate System of 1927, North Dakota North and South zones, meters
 - d. Universal Transverse Mercator of 1983, Zone 13 North, meters
 - e. Universal Transverse Mercator of 1983, Zone 14 North, meters
6. **Linear Unit:** Originally the NGS required the meter to be the defining unit for SPCS2022. However, through feedback and analysis, NGS now requires the international foot as the defining unit for false northings and eastings and it is also the intended working unit of the projected coordinates. This document was created between policy decisions and references both meter and international feet.

7. **Projected coordinate values:** NDCRS coordinates were specified that northings and eastings are not equal everywhere within a zone and coordinate values are always positive in a zone. Specifically:
 - a. Easting coordinates are unique for each zone.
 - b. Northings are not unique for each zone. TM projections have a value of zero. Northings for LCC projections are the same based on whether the zone falls in the north or south part of the state.

Table 1 shows the final design parameters for North Dakota's 16 zones including number of counties in each zone, zone number and zone code for quick reference. All projections are referenced to NAD 83 as the geodetic datum. Six of the 16 zones are TM projections with a false northing of zero at their origin latitude. The origin latitude for TM zones 1-3 and 6 is 46°30'00"N and 45°45'00"N for zones 15 and 16. All of the other ten zones are one-parallel LCC projections where the origin latitude is also the standard parallel.

Table 2 provides a cross-reference showing each of the 53 North Dakota counties and corresponding zone.

Figure 1 shows linear map projection distortion at the topographical surface for all 16 zones and overall statewide performance statistics. The goal of 95% of the state having ± 20 ppm distortion was achieved and 63% has distortion with ± 10 ppm while 99% of the state is within ± 30 ppm. Mean distortion is only slightly negative, -1.0 ppm attributed to large, broad high areas notable in the Turtle Mountains.

Figure 2 graphically shows each zone centroid to illustrate the expected magnitude and difference between NDCRS coordinates in each zone. Zone centroids are expressed in meters and when converted to international feet the beginning of the easting coordinate is equivalent to the zone number.

Appendix A provides distortion maps for each of the 16 zones showing projection parameters and distortion at a 6-mile buffer around the entire zone. These maps will be available on the NDDOT website for printing on 11" x 17" tabloid paper at the scale at which they were created.

Appendix B provides distortion and ellipsoid height values for incorporated places in Table B1.

Appendix C Equations for computing projection grid point scale factors.

Table 1. Defining parameters for the 16 zones of the NDCRS – International feet

- All zones are referenced to the North American Datum of 1983 and based on the Geodetic Reference System of 1980 ellipsoid.
- Projection types: “TM” = Transverse Mercator; “LC” = Lambert Conformal Conic.
- For LC projections, the origin latitude is the standard parallel or projection axis.

| Zone Number | Zone Code | Zone Abrev. | Zone Name | Number of Counties | Projection Type | Origin Latitude | | Origin Longitude | | Projection Axis Scale | False Northing (ift) | False Easting (ift) | False Northing (m) | False Easting (m) |
|-------------|-----------|-------------|-------------|--------------------|-----------------|-----------------|----------------|------------------|----------------|-----------------------|----------------------|---------------------|--------------------|-------------------|
| | | | | | | deg-min | decimal degree | deg-min | decimal degree | | | | | |
| 1 | 381001 | ND_WLS | Williston | 3 | TM | 46° 30' N | 46.5000° | 103° 27' W | -103.4500° | 1.000 092 | 0 | 1,500,000 | 0 | 457,200 |
| 2 | 381002 | ND_NT | New Town | 2 | TM | 46° 30' N | 46.5000° | 102° 27' W | -102.4500° | 1.000 094 | 0 | 2,500,000 | 0 | 762,000 |
| 3 | 381003 | ND_MNT | Minot | 2 | TM | 46° 30' N | 46.5000° | 101° 27' W | -101.4500° | 1.000 080 | 0 | 3,625,000 | 0 | 1,104,900 |
| 4 | 381004 | ND_BOT | Bottineau | 4 | LC | 48° 36' N | 48.6000° | 99° 42' W | -99.7000° | 1.000 066 | 500,000 | 4,500,000 | 152,400 | 1,371,600 |
| 5 | 381005 | ND_DL | Devils Lake | 5 | LC | 48° 09' N | 48.1500° | 99° 27' W | -99.4500° | 1.000 056 | 500,000 | 5,500,000 | 152,400 | 1,676,400 |
| 6 | 381006 | ND_GF | Grand Forks | 5 | TM | 46° 30' N | 46.5000° | 97° 24' W | -97.4000° | 1.000 034 | 0 | 6,500,000 | 0 | 1,981,200 |
| 7 | 381007 | ND_BEU | Beulah | 3 | LC | 47° 27' N | 47.4500° | 101° 51' W | -101.8500° | 1.000 090 | 500,000 | 7,500,000 | 152,400 | 2,286,000 |
| 8 | 381008 | ND_CAR | Carrington | 5 | LC | 47° 33' N | 47.5500° | 99° 18' W | -99.3000° | 1.000 072 | 500,000 | 8,500,000 | 152,400 | 2,590,800 |
| 9 | 381009 | ND_DKS | Dickinson | 3 | LC | 47° 00' N | 47.0000° | 103° 03' W | -103.0500° | 1.000 112 | 375,000 | 9,500,000 | 114,300 | 2,895,600 |
| 10 | 381010 | ND_BIS | Bismarck | 4 | LC | 46° 48' N | 46.8000° | 100° 45' W | -100.7500° | 1.000 081 | 375,000 | 10,500,000 | 114,300 | 3,200,400 |
| 11 | 381011 | ND_JMT | Jamestown | 2 | LC | 46° 57' N | 46.9500° | 98° 36' W | -98.6000° | 1.000 067 | 375,000 | 11,500,000 | 114,300 | 3,505,200 |
| 12 | 381012 | ND_BOW | Bowman | 4 | LC | 46° 18' N | 46.3000° | 103° 00' W | -103.0000° | 1.000 124 | 375,000 | 12,500,000 | 114,300 | 3,810,000 |
| 13 | 381013 | ND_CB | Cannon Ball | 2 | LC | 46° 18' N | 46.3000° | 101° 18' W | -101.3000° | 1.000 093 | 375,000 | 13,500,000 | 114,300 | 4,114,800 |
| 14 | 381014 | ND_LIN | Linton | 3 | LC | 46° 18' N | 46.3000° | 99° 51' W | -99.8500° | 1.000 082 | 375,000 | 14,375,000 | 114,300 | 4,381,500 |
| 15 | 381015 | ND_OAK | Oakes | 2 | TM | 45° 45' N | 45.7500° | 98° 18' W | -98.3000° | 1.000 061 | 0 | 15,375,000 | 0 | 4,686,300 |
| 16 | 381016 | ND_FAR | Fargo | 4 | TM | 45° 45' N | 45.7500° | 97° 12' W | -97.2000° | 1.000 032 | 0 | 16,375,000 | 0 | 4,991,100 |

Table 2. North Dakota Counties and corresponding NDCRS zone.

| County | Zone | Zone No. | County | Zone | Zone No. | County | Zone | Zone No. | County | Zone | Zone No. |
|-----------|-------------|----------|---------------|-------------|----------|-----------|-------------|----------|----------|-------------|----------|
| Adams | Bowman | 12 | Emmons | Linton | 14 | McLean | Beulah | 7 | Sargent | Fargo | 16 |
| Barnes | Jamestown | 11 | Foster | Carrington | 8 | Mercer | Beulah | 7 | Sheridan | Carrington | 8 |
| Benson | Devils Lake | 5 | Golden Valley | Dickinson | 9 | Morton | Bismarck | 10 | Sioux | Cannon Ball | 13 |
| Billings | Dickinson | 9 | Grand Forks | Grand Forks | 6 | Mountrail | New Town | 2 | Slope | Bowman | 12 |
| Bottineau | Bottineau | 4 | Grant | Cannon Ball | 13 | Nelson | Devils Lake | 5 | Stark | Dickinson | 9 |
| Bowman | Bowman | 12 | Griggs | Carrington | 8 | Oliver | Bismarck | 10 | Steele | Grand Forks | 6 |
| Burke | New Town | 2 | Hettinger | Bowman | 12 | Pembina | Grand Forks | 6 | Stutsman | Jamestown | 11 |
| Burleigh | Bismarck | 10 | Kidder | Bismarck | 10 | Pierce | Devils Lake | 5 | Towner | Bottineau | 4 |
| Cass | Fargo | 16 | LaMoure | Oakes | 15 | Ramsey | Devils Lake | 5 | Traill | Grand Forks | 6 |
| Cavalier | Bottineau | 4 | Logan | Linton | 14 | Ransom | Fargo | 16 | Walsh | Grand Forks | 6 |
| Dickey | Oakes | 15 | McHenry | Devils Lake | 5 | Renville | Minot | 3 | Ward | Minot | 3 |
| Divide | Williston | 1 | McIntosh | Linton | 14 | Richland | Fargo | 16 | Wells | Carrington | 8 |
| Dunn | Beulah | 7 | McKenzie | Williston | 1 | Rolette | Bottineau | 4 | Williams | Williston | 1 |
| Eddy | Carrington | 8 | | | | | | | | | |

North Dakota Coordinate Reference System

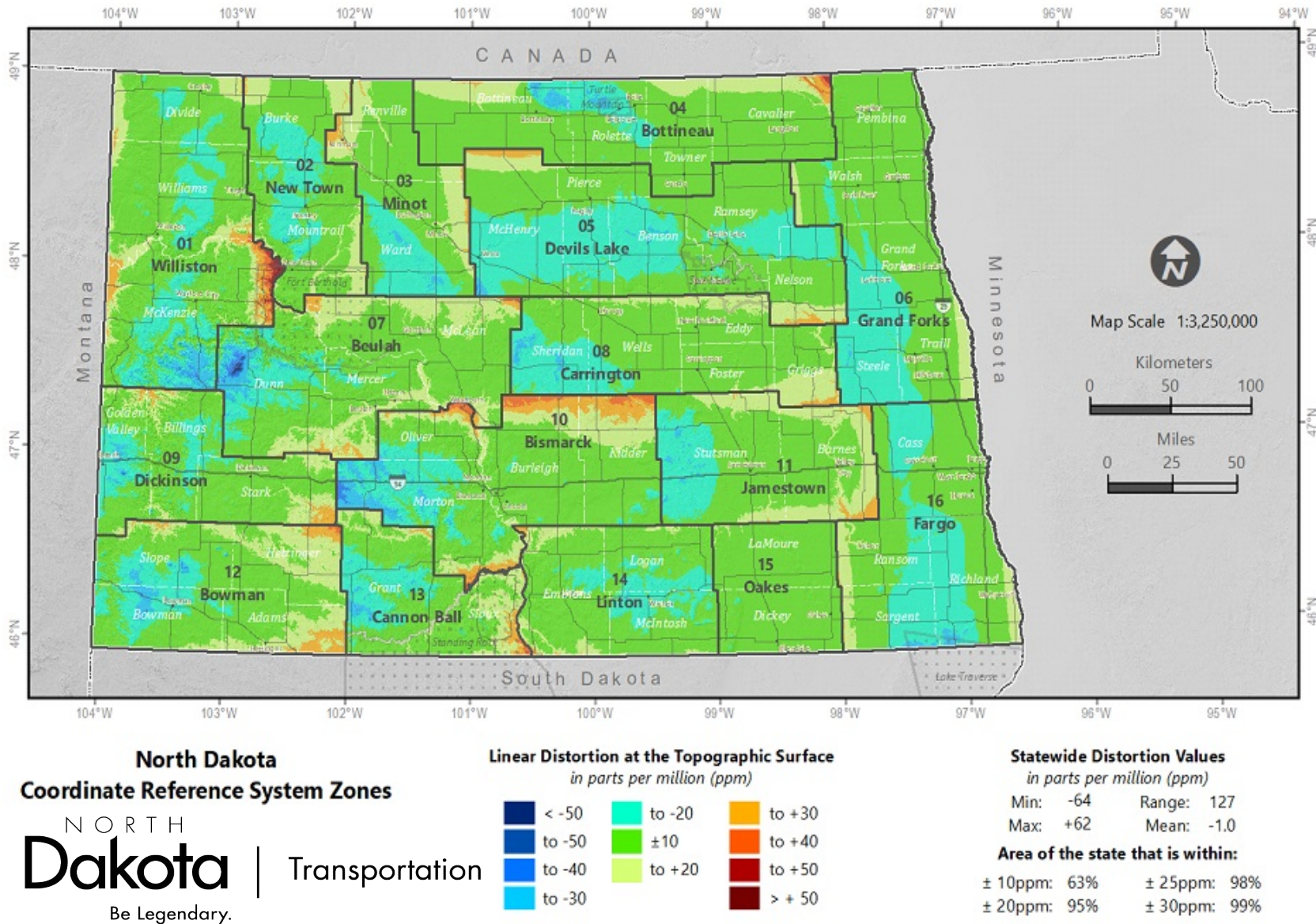


Figure 1. Linear distortion map for NDCRS Zones with statewide statistics.

Projected Coordinate Values

To illustrate the expected magnitude and difference between NDCRS coordinates in each zone, projected coordinates for the centroid of each zone are provided in Figure 2. It should be noted that the centroid coordinates have been rounded to exactly the nearest whole arc-second divisible by 3. This provides an exact decimal degree number to 4 decimal places of precision to serve as a check on NDCRS coordinate computations in computer software. This data is provided in Table 3 which includes the ellipsoid height rounded to the nearest whole meter. For additional computation checks, Table 4 provides the NDCRS coordinate values for each zone centroid point in both meters and international feet along with the linear distortion in ppm, the combined grid scale factor, and convergence angle. When converted to feet, the beginning of the easting coordinate value is equivalent to the zone number.

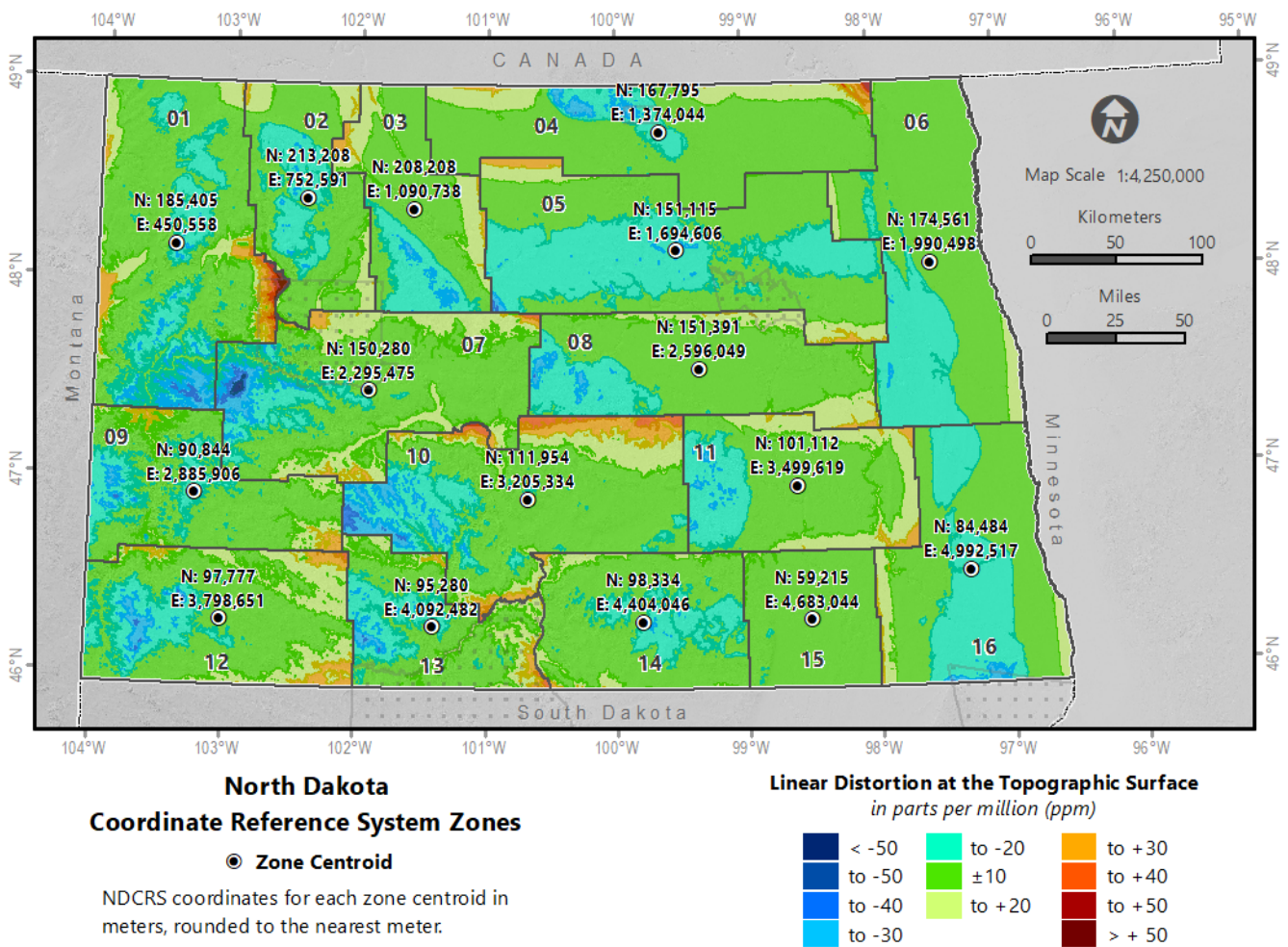


Figure 2. Projected centroid coordinates for NDCRS zones (rounded to nearest meter)

Table 3. Zone centroid coordinates. Latitude and longitude values are to the exact whole arc second and ellipsoid heights are to the whole meter.

| Zone | Zone Name | degrees/minutes/seconds | | | decimal degrees | | | Ellipsoid Height (m) | Northing (m) | Easting (m) | Northing (ift) | Easting (ift) |
|------|-------------|-------------------------|----------------|----------------|-----------------|-----------|-----------|----------------------|--------------|--------------|----------------|---------------|
| | | Latitude | Longitude | | Latitude | Longitude | | | | | | |
| | | | East | West | | East | West | | | | | |
| 1 | Williston | 48° 10' 03" N | 256° 33' 27" E | 103° 26' 33" W | 48.1675 | 256.5575 | -103.4425 | 654 | 185,405.30 | 450,557.93 | 608,285.11 | 1,478,208.42 |
| 2 | New Town | 48° 25' 03" N | 257° 35' 06" E | 102° 24' 54" W | 48.4175 | 257.585 | -102.415 | 697 | 213,207.85 | 752,590.99 | 699,500.83 | 2,469,130.54 |
| 3 | Minot | 48° 22' 21" N | 258° 25' 30" E | 101° 34' 30" W | 48.3725 | 258.425 | -101.575 | 484 | 208,207.52 | 1,090,738.44 | 683,095.53 | 3,578,538.19 |
| 4 | Bottineau | 48° 45' 36" N | 260° 21' 18" E | 99° 38' 42" W | 48.76 | 260.355 | -99.645 | 515 | 167,795.22 | 1,374,044.03 | 550,509.25 | 4,508,018.48 |
| 5 | Devils Lake | 48° 09' 36" N | 260° 28' 39" E | 99° 31' 21" W | 48.16 | 260.4775 | -99.5225 | 475 | 151,114.54 | 1,694,606.11 | 495,782.60 | 5,559,731.34 |
| 6 | Grand Forks | 48° 04' 12" N | 262° 28' 21" E | 97° 31' 39" W | 48.07 | 262.4725 | -97.5275 | 254 | 174,560.74 | 1,990,497.82 | 572,705.84 | 6,530,504.67 |
| 7 | Beulah | 47° 27' 09" N | 258° 05' 24" E | 101° 54' 36" W | 47.4525 | 258.09 | -101.91 | 564 | 150,279.72 | 2,295,474.90 | 493,043.70 | 7,531,085.64 |
| 8 | Carrington | 47° 33' 45" N | 260° 38' 51" E | 99° 21' 09" W | 47.5625 | 260.6475 | -99.3525 | 456 | 151,391.21 | 2,596,048.88 | 496,690.31 | 8,517,220.72 |
| 9 | Dickinson | 46° 55' 03" N | 256° 45' 54" E | 103° 14' 06" W | 46.9175 | 256.765 | -103.235 | 790 | 90,844.08 | 2,885,906.42 | 298,044.89 | 9,468,196.91 |
| 10 | Bismarck | 46° 54' 27" N | 259° 19' 12" E | 100° 40' 48" W | 46.9075 | 259.32 | -100.68 | 561 | 111,953.91 | 3,205,333.54 | 367,302.85 | 10,516,186.16 |
| 11 | Jamestown | 46° 57' 36" N | 261° 23' 42" E | 98° 36' 18" W | 46.96 | 261.395 | -98.605 | 430 | 101,111.79 | 3,499,619.41 | 331,731.58 | 11,481,690.98 |
| 12 | Bowman | 46° 16' 48" N | 256° 58' 57" E | 103° 01' 03" W | 46.28 | 256.9825 | -103.0175 | 819 | 97,776.73 | 3,798,651.08 | 320,789.81 | 12,462,766.01 |
| 13 | Cannon Ball | 46° 15' 27" N | 258° 36' 09" E | 101° 23' 51" W | 46.2575 | 258.6025 | -101.3975 | 643 | 95,280.02 | 4,092,481.74 | 312,598.50 | 13,426,777.37 |
| 14 | Linton | 46° 17' 06" N | 260° 12' 09" E | 99° 47' 51" W | 46.285 | 260.2025 | -99.7975 | 594 | 98,333.85 | 4,404,046.22 | 322,617.61 | 14,448,970.55 |
| 15 | Oakes | 46° 16' 57" N | 261° 28' 48" E | 98° 31' 12" W | 46.2825 | 261.48 | -98.52 | 427 | 59,215.38 | 4,683,043.98 | 194,276.20 | 15,364,317.52 |
| 16 | Fargo | 46° 30' 36" N | 262° 42' 09" E | 97° 17' 51" W | 46.51 | 262.7025 | -97.2975 | 292 | 84,484.25 | 4,992,516.78 | 277,179.31 | 16,379,648.23 |

Table 4. Computed values at zone centroids. These values are provided for computation checks.

| Zone | Zone Name | Northing (m) | Easting (m) | Northing (ift) | Easting (ift) | Linear Distortion (ppm) | Combined Factor | Convergence angle (DMS) |
|------|-------------|--------------|----------------|----------------|-----------------|-------------------------|-----------------|-------------------------|
| 1 | Williston | 185,405.3030 | 450,557.9273 | 608,285.1149 | 1,478,208.4230 | -10.4955 | 0.999 989 5045 | +0°00'20.12" |
| 2 | New Town | 213,207.8539 | 752,590.9874 | 699,500.8332 | 2,469,130.5361 | -15.1524 | 0.999 984 8476 | +0°01'34.25" |
| 3 | Minot | 208,207.5187 | 1,090,738.4403 | 683,095.5337 | 3,578,538.1900 | 5.1982 | 1.000 005 1982 | -0°05'36.37" |
| 4 | Bottineau | 167,795.2187 | 1,374,044.0316 | 550,509.2478 | 4,508,018.4763 | -10.8166 | 0.999 989 1834 | +0°02'28.52" |
| 5 | Devils Lake | 151,114.5377 | 1,694,606.1110 | 495,782.6039 | 5,559,731.3353 | -18.4295 | 0.999 981 5705 | -0°03'14.42" |
| 6 | Grand Forks | 174,560.7386 | 1,990,497.8237 | 572,705.8355 | 6,530,504.6711 | -4.7003 | 0.999 995 2997 | -0°05'41.48" |
| 7 | Beulah | 150,279.7196 | 2,295,474.9034 | 493,043.6994 | 7,531,085.6409 | 1.5986 | 1.000 001 5986 | -0°02'39.12" |
| 8 | Carrington | 151,391.2068 | 2,596,048.8752 | 496,690.3111 | 8,517,220.7191 | 0.5505 | 1.000 000 5505 | -0°02'19.46" |
| 9 | Dickinson | 90,844.0822 | 2,885,906.4171 | 298,044.8890 | 9,468,196.9065 | -10.7992 | 0.999 989 2008 | -0°08'07.08" |
| 10 | Bismarck | 111,953.9088 | 3,205,333.5418 | 367,302.8503 | 10,516,186.1609 | -5.1812 | 0.999 994 8188 | +0°03'03.70" |
| 11 | Jamestown | 101,111.7863 | 3,499,619.4108 | 331,731.5822 | 11,481,690.9803 | -0.3875 | 0.999 999 6125 | -0°00'13.15" |
| 12 | Bowman | 97,776.7336 | 3,798,651.0791 | 320,789.8083 | 12,462,766.0076 | -4.3276 | 0.999 995 6724 | -0°00'45.55" |
| 13 | Cannon Ball | 95,280.0225 | 4,092,481.7412 | 312,598.4991 | 13,426,777.3662 | -7.5239 | 0.999 992 4761 | -0°04'13.76" |
| 14 | Linton | 98,333.8480 | 4,404,046.2242 | 322,617.6115 | 14,448,970.5519 | -11.0820 | 0.999 988 9180 | +0°02'16.64" |
| 15 | Oakes | 59,215.3846 | 4,683,043.9796 | 194,276.1962 | 15,364,317.5184 | -2.4051 | 0.999 997 5949 | -0°09'32.42" |
| 16 | Fargo | 84,484.2543 | 4,992,516.7801 | 277,179.3119 | 16,379,648.2288 | -13.0849 | 0.999 986 9151 | -0°04'14.65" |

Using the NDCRS requires that its parameters be entered into software by the software user or the manufacturer. The possibility exists for system parameters to be entered incorrectly and not all software is programmed to perform calculations the same way resulting in incorrect results. For this reason, computation checks should be done to ensure correctness. If the latitude and longitude values for a zone listed in Table 3 are entered into software the results should show the NDCRS values shown. Conversely, if the NDCRS coordinates are entered they should produce the latitude and longitude values shown. If the software does not produce the value shown in Table 3, check to ensure all parameters have been entered correctly, the units are correct, and the datum has been properly defined with the correct reference ellipsoid and no datum transformation. Datum transformations are a common cause of coordinate computation errors. To duplicate the coordinates in Table 3, the latitude and longitude as shown must be used in the projection algorithms and cannot be transformed before projecting. The reverse is also true – the latitude and longitude generated by de-projecting the NDCRS coordinates must not be transformed after de-projection.

The NDCRS projected coordinates are defined such that they differ from other existing coordinate systems defined in North Dakota. These systems include State Plane Coordinate System of 2022 North Dakota statewide zone, State Plane North Dakota both North and South zones based in both NAD 83 defined by meters and NAD 27 defined by US survey foot; UTM 83 zones 13 North and 14 North defined by meters. To illustrate, the horizontal difference was calculated from the zone centroids using the NDCRS coordinates listed in table 3 and 4 to the various other coordinate systems previously identified and shown in Table 5.

Table 5. Differences in coordinate values between NDCRS and other North Dakota coordinate systems.

Differences are given in meters, rounded to the nearest whole meter.

| NDCRS Zone No. | NDCRS Zone Name | SPCS 27 and 83 Zone | UTM Zone | Horizontal difference from NDCRS (meters) | | |
|----------------|-----------------|---------------------|----------|---|-----------|-----------|
| | | | | SPCS 27 | SPCS 83 | UTM |
| 1 | Williston | N | 13 | 78,820 | 86,345 | 5,153,335 |
| 2 | New Town | N | 13 | 289,737 | 299,205 | 5,153,091 |
| 3 | Minot | N | 14 | 563,439 | 573,026 | 5,211,616 |
| 4 | Bottineau | N | 14 | 702,115 | 711,744 | 5,313,686 |
| 5 | Devils Lake | N | 14 | 1,012,489 | 1,022,122 | 5,327,848 |
| 6 | Grand Forks | N | 14 | 1,160,543 | 1,170,169 | 5,332,446 |
| 7 | Beulah | S | 14 | 1,792,841 | 1,802,470 | 5,492,197 |
| 8 | Carrington | N | 14 | 1,902,112 | 1,911,736 | 5,539,158 |
| 9 | Dickinson | S | 13 | 2,485,094 | 2,494,723 | 5,580,981 |
| 10 | Bismarck | S | 14 | 2,609,543 | 2,619,175 | 5,820,444 |
| 11 | Jamestown | S | 14 | 2,746,132 | 2,755,766 | 5,901,279 |
| 12 | Bowman | S | 13 | 3,383,115 | 3,392,746 | 5,932,165 |
| 13 | Cannon Ball | S | 14 | 3,552,168 | 3,561,799 | 6,290,495 |
| 14 | Linton | S | 14 | 3,740,391 | 3,750,025 | 6,403,334 |
| 15 | Oakes | S | 14 | 3,920,848 | 3,930,484 | 6,546,571 |
| 16 | Fargo | S | 14 | 4,137,196 | 4,146,834 | 6,686,343 |

NDCRS Design Methodology and Results

NDCRS design was completed using a digital elevation model derived from the United States Geological Survey's 3D Elevation Program at a resolution of 1 arc-second which translates to approximately 300 ft on the ground. The 3DEP provides the North American Vertical Datum of Datum of 1988 orthometric heights. The NAVD 88 heights were converted to NAD 83 ellipsoid heights by adding the NGS hybrid geoid model GEOID18 interpolated to the same resolution as the DEM.

Map projection linear distortion is caused by change in topographic height and Earth curvature due to the projected surface departing from the topographic surface. Total distortion is based on height and Earth curvature making it near impossible to have a singular design height for large areas. The reason this is particularly true for large areas is that the contribution of Earth curvature to total distortion increases as the size of the area increases. Consequently, topographic ellipsoid height was not used directly as a design parameter.

A ± 400 -foot change in height causes approximately ± 20 ppm of distortion. Figure 3 shows the topographic ellipsoid heights for North Dakota. The state has a range in topographic height of about 2,800 feet. This range in topographic height alone would require at least four zones to keep distortion within ± 20 ppm. However, curvature of the Earth also needs to be taken into consideration where a zone width of 70 miles perpendicular to the projection axis will cause about ± 20 ppm, requiring North Dakota's east-west length of approximately 350 miles to create about six zones. By combining topographic height and curvature of the earth it would suggest that about 20 zones are needed to achieve ± 20 ppm distortion across North Dakota. In keeping with the project goal to have zones follow county boundaries while achieving the fewest number of zones possible, the NDCRS was developed with 16 zones.

Design of the NDCRS:

- 1) A review was completed of linear distortion by county for SPCS 83 using the NDDOT county coordinate conversion factors. Preliminary zones by county groupings was created taking into consideration large population centers and transportation corridors. The preliminary design included 15 zones. Initial design and analyses was performed on these zones, resulting in four of the zones not achieving the objective of 95% of the zone having ± 20 ppm distortion. Alternative zones were created for these areas along with options for the east and west ends of the state. These design alternatives were evaluated resulting in further refinement in the next design phase.
- 2) Based on initial feedback and detailed review of preliminary designs, 16 final zones were established across the state. Projection determination depends on topographical height and slope. Zones that are longer north and south have less distortion using a Transverse Mercator projection while zones that are longer east and west perform better with a Lambert Conformal Conic projection. The final design included six TM zones and ten LCC zones. Design and analyses were based on a one-kilometer grid of 183,096 ellipsoid height points across the state, 495 city and town points, and 26,548 population points based on the block centroids from the 2010 decennial Census.
- 3) Final design maps are in Appendix A. A 6-mile buffer extending past the zone boundaries is included to provide visual guidance for projects that occur near or cross over zone boundaries that might benefit from lower distortion using the projection from an adjacent zone.

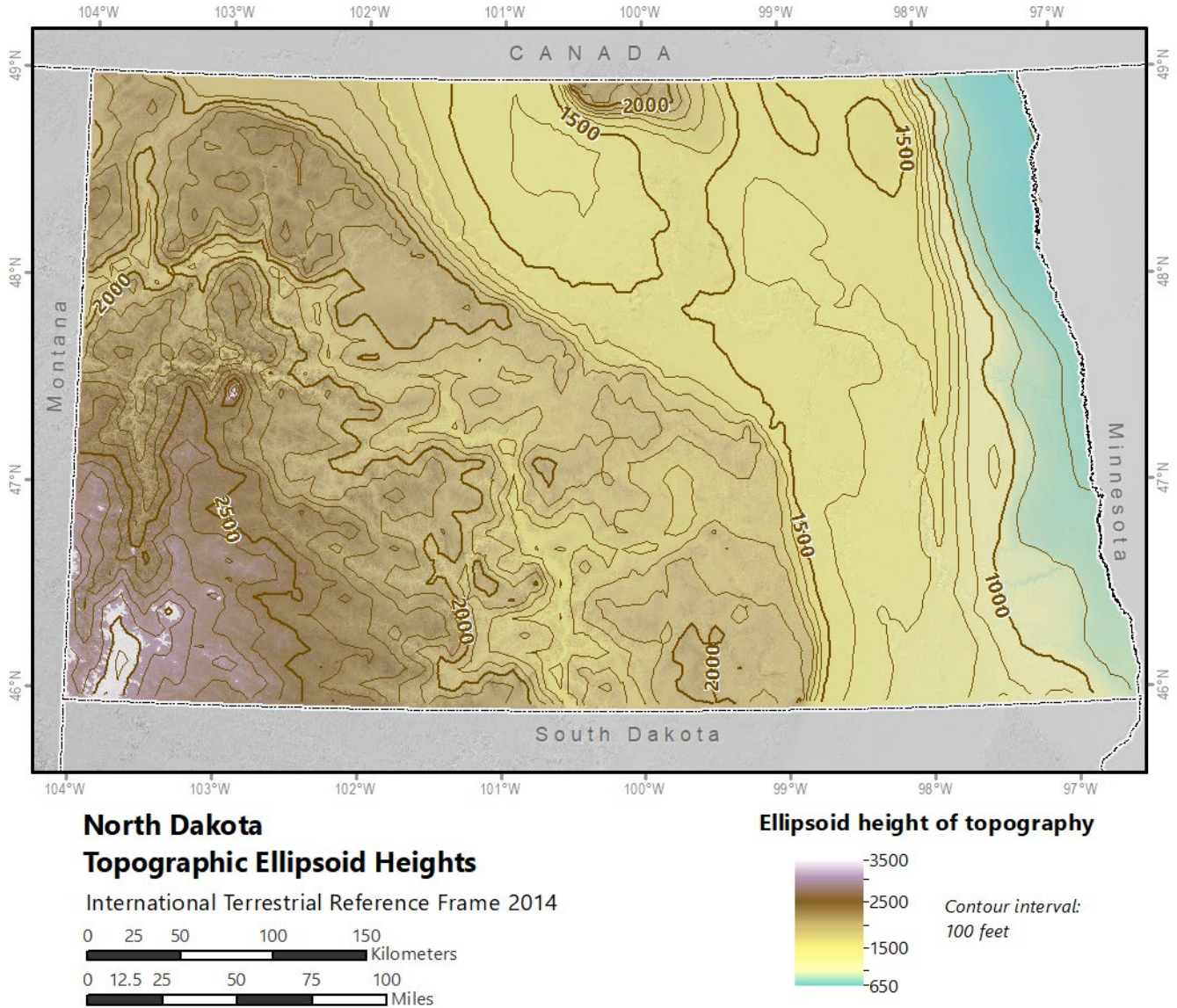


Figure 3. Topographic Ellipsoid Heights in North Dakota

Final NDCRS distortion and ellipsoid height statistics for each zone and the entire state are given in Table 6. The distortion and ellipsoid height statistics were computed from the 3 arc-second distortion and ellipsoid rasters. Table 6 gives distortion statistics for minimum, maximum, range, mean, standard deviation and ellipsoid height. Slightly over 95% of the state has distortion within ± 20 ppm which meets the stated objective. Five of the 16 zones, Williston (91.0%), Beulah (93.4%), Bismarck (85.1%), Bowman (92.5%), and Cannon Ball (94.5%) have less than 95% of the zone with ± 20 ppm. The smallest zone, Oakes, has complete coverage with distortion at ± 15 ppm. Grand Forks, among one of the largest zones, is almost completely (99.9%) within ± 20 ppm. Half of the zones have complete coverage at ± 30 ppm.

The highest distortion values in the state occur in the Williston (+62.4 ppm) and Beulah (-64.4 ppm). In Williston this occurs along the Missouri River at the far east edge of the zone. In Beulah it is the Killdeer Mountains along the western edge of the zone causing the most distortion. Adjacent zones provide lower distortion in these areas if needed.

Table 6 shows the ellipsoid height range of 2,770 ft. from 655 ft to 3,425 ft. The range for each zone vary and the highest range occurs in Beulah at 1,697 ft due to the Killdeer Mountains.

Table 6. Linear distortion and ellipsoid height statistics for each zone and statewide.

Ellipsoid heights are shown in international feet. Table 3 provides ellipsoid height values in international feet.

| Zone | Zone Name | Distortion in parts per million (ppm) | | | | | Percent of Zone within stated PPM | | | Ellipsoid Height (ift) | | |
|------------------|-------------|---------------------------------------|-------------|--------------|-------------|-------------|-----------------------------------|-------------|-------------|------------------------|--------------|--------------|
| | | Min | Max | Range | Mean | St. Dev. | ±10 ppm | ±20 ppm | ±30 ppm | Min | Max | Range |
| 1 | Williston | -35.0 | 62.4 | 97.4 | -1.4 | 12.3 | 61.8 | 91.0 | 98.0 | 1,769 | 2,712 | 943 |
| 2 | New Town | -23.3 | 22.9 | 46.2 | -1.1 | 10.1 | 59.7 | 98.9 | 100.0 | 1,717 | 2,490 | 774 |
| 3 | Minot | -24.8 | 29.0 | 53.8 | 0.5 | 11.2 | 57.5 | 96.2 | 100.0 | 1,417 | 2,294 | 877 |
| 4 | Bottineau | -31.1 | 49.1 | 80.2 | 0.0 | 9.6 | 74.6 | 95.8 | 99.4 | 862 | 2,444 | 1,581 |
| 5 | Devils Lake | -34.5 | 31.5 | 66.0 | -5.9 | 9.1 | 53.2 | 97.3 | 99.9 | 1,194 | 2,101 | 907 |
| 6 | Grand Forks | -21.1 | 21.5 | 42.6 | -3.5 | 7.6 | 71.1 | 99.9 | 100.0 | 655 | 1,569 | 915 |
| 7 | Beulah | -64.4 | 40.8 | 105.1 | 1.9 | 11.4 | 61.2 | 93.4 | 98.9 | 1,534 | 3,231 | 1,697 |
| 8 | Carrington | -31.7 | 30.1 | 61.8 | 1.0 | 10.0 | 65.7 | 97.6 | 100.0 | 1,181 | 2,172 | 991 |
| 9 | Dickinson | -46.3 | 31.9 | 78.2 | -1.0 | 9.8 | 67.6 | 96.3 | 99.9 | 1,928 | 3,367 | 1,439 |
| 10 | Bismarck | -37.7 | 42.7 | 80.4 | 1.0 | 13.5 | 57.0 | 85.1 | 96.6 | 1,488 | 2,485 | 997 |
| 11 | Jamestown | -25.0 | 31.7 | 56.7 | 2.0 | 10.8 | 55.0 | 96.6 | 100.0 | 1,051 | 2,058 | 1,007 |
| 12 | Bowman | -38.5 | 38.7 | 77.2 | 1.5 | 11.6 | 55.0 | 92.5 | 99.7 | 2,104 | 3,425 | 1,322 |
| 13 | Cannon Ball | -34.5 | 38.9 | 73.4 | 0.6 | 11.4 | 56.0 | 94.5 | 99.4 | 1,531 | 2,758 | 1,227 |
| 14 | Linton | -22.5 | 27.9 | 50.4 | -2.1 | 8.3 | 72.6 | 99.1 | 100.0 | 1,524 | 2,193 | 669 |
| 15 | Oakes | -14.0 | 13.6 | 27.5 | 1.0 | 3.9 | 99.0 | 100.0 | 100.0 | 1,188 | 2,161 | 973 |
| 16 | Fargo | -40.8 | 22.9 | 63.7 | -5.7 | 8.9 | 56.6 | 98.2 | 100.0 | 750 | 1,636 | 885 |
| Statewide | | -64.4 | 62.4 | 126.8 | -1.0 | 10.7 | 62.8 | 95.3 | 99.3 | 655 | 3,425 | 2,770 |

Linear distortion and ellipsoid heights are also provided in Appendix B for 94 incorporated places in North Dakota with an estimated population of 500 or more based on Census 2019 population estimates. For locations not included in Table B-1, distortion can be estimated from the maps provided in Appendix A. If the distortion rasters are available distortion can be obtained at any location in the state using the 3 arc-second raster for the most accurate values. The table includes the coordinate of the point where the distortion is computed so they can also serve as a distortion computation check.

Using the NDCRS

Defining the NDCRS in Software

The NDCRS was designed using TM and LCC projections which are well defined existing conformal map projection types commonly available in GIS, surveying, and engineering software. Definitions for the NDCRS zones were provided in Esri projection format as deliverables for this project. The *.prj format is a Well-Known Text mark-up language representation of a projected coordinate system, as defined by the Open Geospatial Consortium in 2019. WKT for coordinate reference systems was developed to provide a standardized means of defining coordinate systems in machine-readable format.

Not all geospatial software directly uses WKT representations for defining projected coordinate systems. For software that is not included in the deliverable and that cannot consume the provided WKT representations the projection parameters in Table 1 can be used to manually define NDCRS zones.

In many coordinate system definition formats the latitude and longitude are represented in decimal degrees. When repeating decimal digits occur (ex. $97^{\circ}20'W = -97.333333333333...^{\circ}$) it is recommended that full double-precision be used to represent the number. 16 digits should be used corresponding to 14 decimal places for latitude and longitude less than 100° and 13 decimal places for longitude greater than or equal to 100° . The WKT format should automatically use double precision representation when 16 or more significant digits are provided while defining the parameters. Ten decimal places correspond to about 0.000 04 ft or less which is likely sufficient for most practical applications. It is good practice to define the parameters to full double precision representation to help minimize problems with accumulated round-off error including repeatedly projecting and de-projecting coordinates.

An important part of defining the projection parameters is performing computation checks. Performing checks provides a means for capturing overlooked errors in the parameters and projection algorithms. The centroid coordinates in Table 3 are provided for that purpose. Geospatial software should match the projected coordinates to the precision shown to better than ± 0.0001 ft. You can also check if the northings and eastings in Table 3 are de-projected, they should yield latitude and longitude values that matches those in the table to six decimal places ($\pm 0.000 001''$).

In rare instances, the single-parallel LCC projections used for the NDCRS may be problematic to implement due to software limitations including surveying field software. For such situations, satisfactory alignment with the NDCRS in small areas can usually be achieved using a best-fit planar conformal transformation (affine transformations should not be used because the NDCRS itself is conformal). Such an approach is referred to horizontal calibration or localization in geospatial software. For areas with a maximum dimension of less than about 3 miles differential distortion distribution will usually cause less than 0.05 ft mean alignment error. In these cases the transformation should be based on a minimum of three common points distributed across the extent of the area. Common points are any physical points correctly referenced to NAD 83 that can be observed in the field and for which NDCRS coordinates are available.

Although such an approach can produce acceptable results it should only be applied if the rigorous projection definitions cannot be used and completed by practitioners with sufficient knowledge and skill in using and interpreting such transformations.

Distortion and Ground Distance Computations

Linear distortion, δ , at any point in a projected coordinate system can be computed using the following equation,

$$\delta = k \left(\frac{R}{R+h} \right) - 1$$

where k is the projection grid point scale factor

R is the geometric mean radius of curvature of the GRS-80 ellipsoid

h is the NAD 83 ellipsoid height

Distortion is multiplied by 1,000,000 to obtain parts per million (ppm). Note that if the minus 1 is removed from the equation it becomes the *combined (scale) factor* familiar to surveyors. The quantity in parentheses is the *height (elevation) factor*. The combined scale factor is given for the points listed in table 5.

For greater accuracy, R can be computed at any latitude as

$$R = \frac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi}$$

where a is the semi-major axis of the GRS-80 ellipsoid = 6,378,137 m (exact) \approx 20,925,646.3255 ift

e^2 is the first eccentricity squared of the GRS-80 ellipsoid = 0.0066943800229

φ is the geodetic latitude

Some software may not compute the grid point scale factor, k . Equations for computing k are fairly complex and are a function of position as well as the projection characteristics. These values are given in Appendix C for the TM and LCC projections.

Linear distortion is defined at a point, but practical applications are concerned with how well the projected grid distance matches the true horizontal ground distance between a pair of points. Such evaluations require an accurate method for computing ground distance. Although there is no standard definition for ground distance it can be argued that a reasonable definition is the curved distance parallel to the ellipsoid surface at the mean ellipsoid height of the endpoints. Two relatively simple methods for computing such distance based on this definition are provided below.

One method for computing horizontal ground distance, D , consists of scaling the ellipsoid distance (geodesic) using the average of the ellipsoid heights at the endpoints:

$$D = s \left(1 + \frac{\bar{h}}{R_m} \right)$$

where s is the ellipsoid distance (geodesic)

\bar{h} is the average ellipsoid height of the two points

R_m is the geometric mean radius of curvature at the midpoint latitude of the two points

If software for computing the ellipsoid distance is not available, the NGS Geodetic Tool Kit inverting tools can be used at the following NGS webpage: geodesy.noaa.gov/TOOLS/Inv_Fwd/Inv_Fwd.html

A second method for computing a horizontal ground distance makes use of a GPS vector directly. Neglecting Earth curvature, this distance can be computed using the vector East-Centered, Earth-Fixed Cartesian coordinate deltas,

$$D = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2 - \Delta h^2}$$

where $\Delta X, \Delta Y, \Delta Z$ are the GNSS vector components as ECEF Cartesian coordinate deltas
 Δh is the change in ellipsoid height between vector endpoints

This method can also be used with endpoint coordinates instead of a GNSS vector by converting the latitude, longitude, and ellipsoid heights to X, Y, Z ECEF coordinates and then using the difference in ECEF coordinates. If software is not available for computing ECEF coordinates, the NGS Geodetic Tool Kit *XYZ Conversion* tool can be used for this purpose following the NGS webpage: geodesy.noaa.gov/TOOLS/XYZ/xyz.shtml.

Curvature increases the horizontal ground distance but for distances less than 20 miles, the error is less than 1 part per million which translates to 0.1 ft. The straight-line horizontal distance can be multiplied by the following curvature correction factor C to get the approximate curved horizontal ground distance, where all variables are as defined previously.

$$C = \frac{2R_m \sin^{-1}(D/2R_m)}{D}$$

With the curvature correction, distances of less than 100 miles the error is less than 0.005 ppm, less than 0.003 ft. A mean Earth radius of curvature of 5,865,000 meters can be used for North Dakota or computed using the equation provided earlier.

Although equations have been provided here for accurately computing long ground distances it is important to understand that linear distortion varies continuously along the line between two points. When a ground distance is compared to a projected distance the distortion being evaluated is the average distortion along the line. For long distances greater than a few miles such comparisons are of questionable value since the distortion varies both with the distance and height difference between the endpoints.

Compatibility with the 2022 Terrestrial Reference Frame

NGS is in the process of updating the National Spatial Reference System and will replace NAD 83 with new terrestrial reference frames planned for release sometime in 2025. The TRF referenced to the North American tectonic plate will be called the *North American Terrestrial Reference Frame of 2022* (see Smith et al., 2017). NAVD 88 will also be replaced with a new vertical geopotential datum at the same time although it will have no effect on the terrestrial reference frames or projected coordinate systems.

In North Dakota, NATRF2022 coordinates will differ horizontally from NAD 83 by about 1.4 meters and will be lower than NAD 83 ellipsoid heights by about 0.8 meters depending on location. The GRS-80 ellipsoid will be used for NATRF2022 so the NDCRS projected coordinates will change by the same horizontal amount, and the decrease in ellipsoid height will cause a change in linear distortion by approximately 0.2 ppm. This increase is essentially negligible and will cause the magnitude of distortion to increase slightly in locations where it is positive and decrease slightly where it is negative. Thus, the NDCRS will perform equally well when referenced to NATRF2022.

The NDCRS was developed to be part of the SPCS 2022. The design criteria and parameters were developed in keeping with NGS guidelines for SPCS 2022 as previously outlined.

Concluding Remarks

Summary of NDCRS Design

The North Dakota Coordinate Reference System is a 16-zone system of low-distortion projections that optimally minimize map projection linear distortion at the topographic surface for the entire state of North Dakota. A criterion of ± 20 ppm (± 0.1 ft/mile) distortion was used for design of the NDCRS and was achieved for 95% of the state area.

NDCRS zones consist of 2 to 5 aggregated counties which zone boundaries correspond to county boundaries. As part of the design process a 6-mile buffer was applied around each zone to give options for projects that cross zone boundaries. Due to variations including county distribution, size and shape, distortion near zone boundaries may be greater within a zone than it is outside of an adjacent zone. To achieve the best distortion performance for projects located near zone boundaries it is recommended that the project area be examined prior to selecting the NDCRS zone. This can be done using the maps in Appendix A or by overlaying the project area on the distortion rasters in a GIS application. In some cases a project may be too long to assign to a single zone and it may be better to split the project between two zones rather than use a single zone. These instances should not be a common occurrence and is recommended they be evaluated on a case-by-case basis.

Adoption of the NDCRS by the North Dakota Department of Transportation replaces the previous method of scaling SPCS 83 coordinates to ground on the topographic surface. Since these coordinate systems were based on a modification of SPCS they were not defined using standard projection parameters which made them difficult to use consistently across geospatial platforms (e.g., in surveying, engineering, CAD, and GIS software). In contrast the NDCRS is limited to 16 projected coordinate systems which are rigorously defined using two existing projections, the Transverse Mercator and the (one-parallel) Lambert Conformal Conic which are both supported by a wide range of software platforms. Importantly they are also available for creating standardized projected coordinate system definitions in the European Petroleum Survey Group *Geodetic Parameter Registry* (2020). The EPSG registry is a freely available database of coordinate system definitions that is widely accessed by the software industry and government organizations. The registry provides machine-readable definitions in either or both Well-Known Text and Geography Markup Language formats. The NDCRS will be submitted to the EPSG registry by NGS as a part of SPCS 2022. This will aid in the adoption of the NDCRS into commercial software packages.

LDP coordinate systems have been defined and officially adopted by government agencies throughout the US. Most of these systems will also be included in SPCS 2022. Documents for several of the systems that have been adopted are listed in the references section of this report and available online: *Oregon Coordinate Reference System* (Armstrong, 2017); *Iowa Regional Coordinate System* (Dennis, 2014); *Indiana Geospatial Coordinate System* (Badger, 2016); *Wisconsin Coordinate Reference Systems* (Wisconsin State Cartographer's Office, 2012); *Rocky Mountain Tribal Coordinate Reference System* (Dennis, 2014); *Pima County Coordinate System* (Dennis, 2017); *Kansas Regional Coordinate System* (Dennis, 2017).

References

- Armstrong, M.L., Thomas, J., Bays, K., and Dennis, M.L., 2017. *Oregon Coordinate Reference System Handbook and Map Set*, version 3.01, Oregon Department of Transportation, Geometronics Unit, Salem, Oregon, <ftp://ftp.odot.state.or.us/ORGN/Documents/ocrs_handbook_user_guide.pdf> (Jun 1, 2020)
- Badger, M.G. (lead author), 2016. *Indiana Geospatial Coordinate System Handbook and User Guide*, version 1.05, Indiana Department of Transportation and Land & Aerial Survey Office, Indianapolis, Indiana <www.in.gov/indot/files/InGCS_HandbookUserGuide_20160901.pdf> (Jun 1, 2020)
- Dennis, M.L. (editor), 2014. *Rocky Mountain Tribal Coordinate Reference System Handbook and User Guide*, v1.0, Rocky Mountain Tribal Transportation Association <www.neciusa.com/documents/RMTCRS_Manual.pdf> (Feb 2, 2020)
- Dennis, M.L., 2016. "Ground Truth: Low Distortion Map Projections for Engineering, Surveying, and GIS", *Proceedings of the Pipelines 2016 Conference*, Utility Engineering and Surveying Institute of the American Society of Civil Engineers, July 17–20, 2016 Kansas City, Missouri, USA.
- Dennis, M.L., 2017a. "Ground Truth: Optimized Design of Low Distortion Projections", Alaska Surveying and Mapping Conference, February 14, 2017, Anchorage, Alaska, in the *NGS Presentations Library*, National Oceanic and Atmospheric Administration, National Geodetic Survey, Silver Spring, Maryland <geodesy.noaa.gov/web/science_edu/presentations_library/> (Apr 15, 2020)
- Dennis, M.L., 2017b. *Pima County Coordinate System*, Information Technology Department, GIS Division, Pima County, Tucson, Arizona <webcms.pima.gov/UserFiles/Servers/Server_6/File/Government/Geographic%20Information%20Systems/GIS%20Maps/PCCS/PCCS_report.pdf> (Apr 15, 2020)
- Dennis, M.L., 2017c. *Kansas Regional Coordinate System*, Kansas Department of Transportation, Topeka, Kansas <data.kansasgis.org/catalog/other/KS_LDP/KRCS_report_2017-11-01.pdf> (Apr 15, 2020)
- Dennis, M.L., Miller, N., and Brown, G., 2014. *Iowa Regional Coordinate System Handbook and User Guide*, version 2.10, Iowa Department of Transportation, Ames, Iowa <www.iowadot.gov/rtn/pdfs/IaRCS_Handbook.pdf> (Apr 16, 2020)
- Dewberry, 2012. *National Enhanced Elevation Assessment*, Fairfax, Virginia <www.dewberry.com/services/geospatial/national-enhanced-elevation-assessment> (Apr 16, 2020)
- European Petroleum Survey Group, 2020. "EPSG Geodetic Parameter Dataset", *EPSG Geodetic Parameter Registry*, version 9.8.12, Geodesy Subcommittee of the IOGP Geomatics Committee, International Association of Oil & Gas Producers <www.epsg-registry.org> (Jun 23, 2020)
- Iliffe, J.C. and Lott, R., 2008. *Datums and Map Projections: For Remote Sensing, GIS and Surveying* (2nd edition), Whittles Publishing, United Kingdom.
- National Geodetic Survey, 2016. *GEOID18* <www.ngs.noaa.gov/GEOID/GEOID18/> (Dec 31, 2019)

- National Geospatial-Intelligence Agency, 2014a. *Department of Defense World Geodetic System of 1984: Its Definition and Relationships with Local Geodetic Systems*, version 1.0.0, NGA.STND.0036_1.0.0_WGS8 (National Geospatial-Intelligence Agency Standardization Document) <earth-info.nga.mil/GandG/update/index.php?dir=wgs84&action=wgs84#tab_wgs84-res> (Apr 16, 2020).
- National Geospatial-Intelligence Agency, 2014b. *The Universal Grids and the Transverse Mercator and Polar Stereographic Map Projections*, version 2.0.0, NGA.SIG.0012_2.0.0_UTMUPS (National Geospatial-Intelligence Agency Standardization Document) <earth-info.nga.mil/GandG/update/index.php?dir=coordsys&action=coordsys#tab_coordsys-res> (Apr 16, 2020)
- National Geospatial-Intelligence Agency, 2014c. *Universal Grids and Grid Reference Systems*, version 2.0.0, NGA.STND.0037_2.0.0_GRIDS (National Geospatial-Intelligence Agency Standardization Document) <earth-info.nga.mil/GandG/update/index.php?dir=coordsys&action=coordsys#tab_coordsys-res> (Apr 16, 2020)
- National Oceanic and Atmospheric Administration, 2023 *State Plane Coordinate System of 2022 Policy*. < www.ngs.noaa.gov/SPCS/policy.shtml>
- National Oceanic and Atmospheric Administration, 2023 *Procedures for Design and Modifications of the State Plane Coordinate System of 2022* < www.ngs.noaa.gov/SPCS/policy.shtml>
- Open Geospatial Consortium, 2019. *Geographic information – Well-known text representation of coordinate reference systems*, OGC document: 18-010r7, v2.0.6 <docs.opengeospatial.org/is/18-010r7/18-010r7.html> (Apr 16, 2020)
- Smith, D.A., Roman, D.R., and Hilla, S.A., 2017. "Blueprint for 2022, Part 1: Geometric Coordinates", *NOAA Technical Report NOS NGS 62*, National Oceanic and Atmospheric Administration, National Geodetic Survey, Silver Spring, Maryland <www.ngs.noaa.gov/PUBS_LIB/NOAA_TR_NOS_NGS_0062.pdf> (Apr 16, 2020)
- Snyder, J.P., 1987. *Map Projections — A Working Manual*, U.S. Geological Survey Professional Paper 1395, Washington, D.C. <pubs.er.usgs.gov/publication/pp1395> (Apr 16, 2020)
- Stem, J.E., 1990. *State Plane Coordinate System of 1983, NOAA Manual NOS NGS 5*, U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Geodetic Survey, Rockville, Maryland <www.ngs.noaa.gov/PUBS_LIB/ManualNOSNGS5.pdf> (Dec 31, 2019)
- Van Sickle, J., 2017. *Basic GIS Coordinates* (3rd edition), CRC Press, Boca Raton, Florida
- Wisconsin State Cartographer's Office, 2015. *Wisconsin Coordinate Reference Systems*, 2nd Edition, State Cartographer's Office, Madison, Wisconsin <www.sco.wisc.edu/images/stories/publications/WisCoordRefSys_June2015.pdf> (Apr 15, 2020)

Appendices

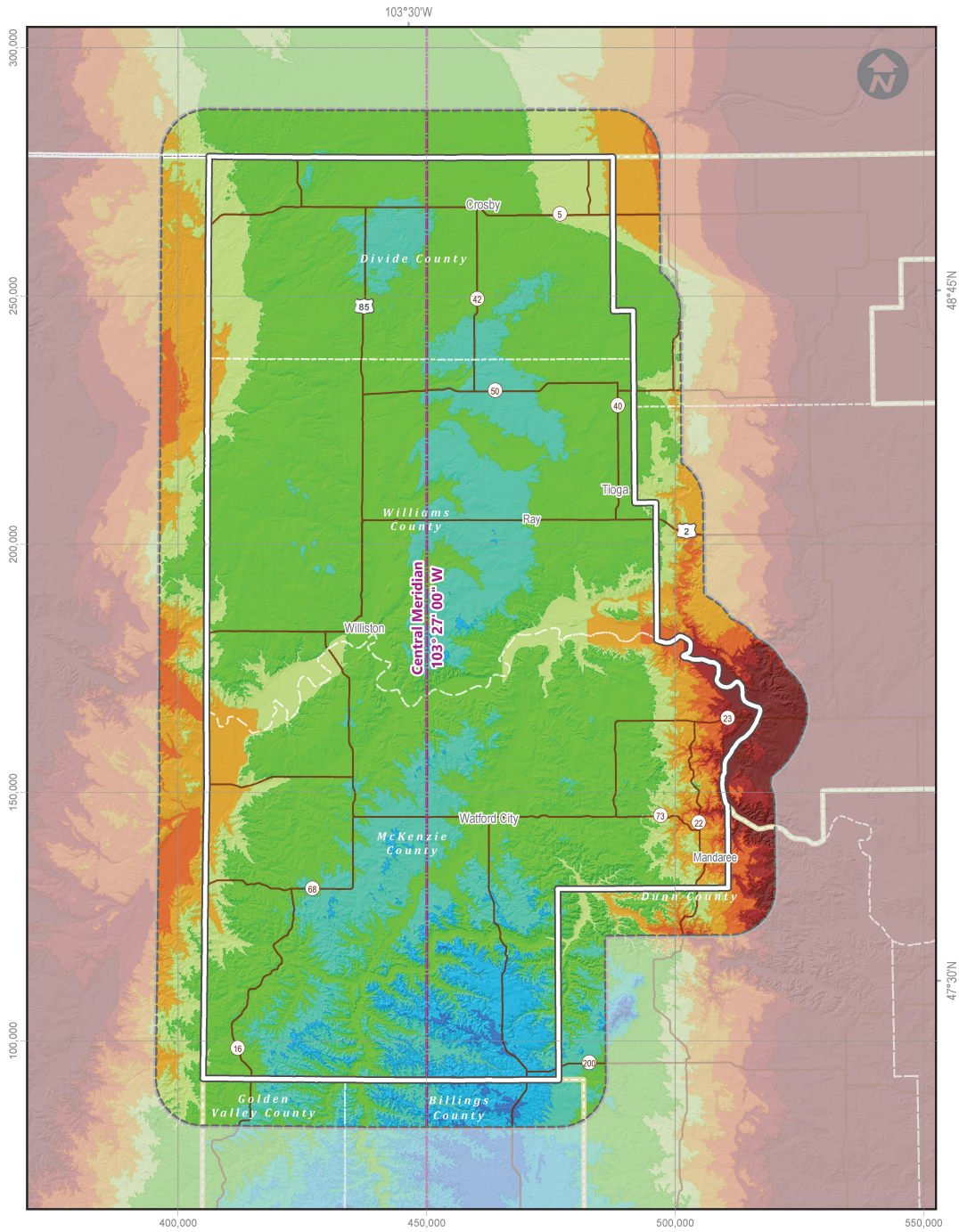
Appendix A: Distortion maps

Appendix B: Distortion and ellipsoid height values for select incorporated places

Appendix C: Equations for computing projection grid point scale factors

Appendix A. Distortion maps

Distortion maps depict the linear distortion in parts per million (ppm) in increments of 10 ppm. The maps in this report have been scaled to fit within the limitations of the page size.



North Dakota Zone 1: Williston

Transverse Mercator

Linear Unit: *ift*
 Latitude of Grid Origin: 46° 30' N
 Central Meridian: 103° 27' W
 False Northing: 0
 False Easting: 1,500,000
 Central Meridian Scale: 1.000092

Linear Distortion at the Topographic Surface
in parts per million (ppm)

| | | | | | | | |
|--|--------|--|--------|--|--------|--|--------|
| | < -50 | | to -30 | | to +20 | | to +50 |
| | to -50 | | to -20 | | to +30 | | > +50 |
| | to -40 | | ±10 | | to +40 | | |

LDP Zone
 6-mile Buffer
 Road
 Other Zone
 County

Scale 1:750,000

0

5

10

Miles

0

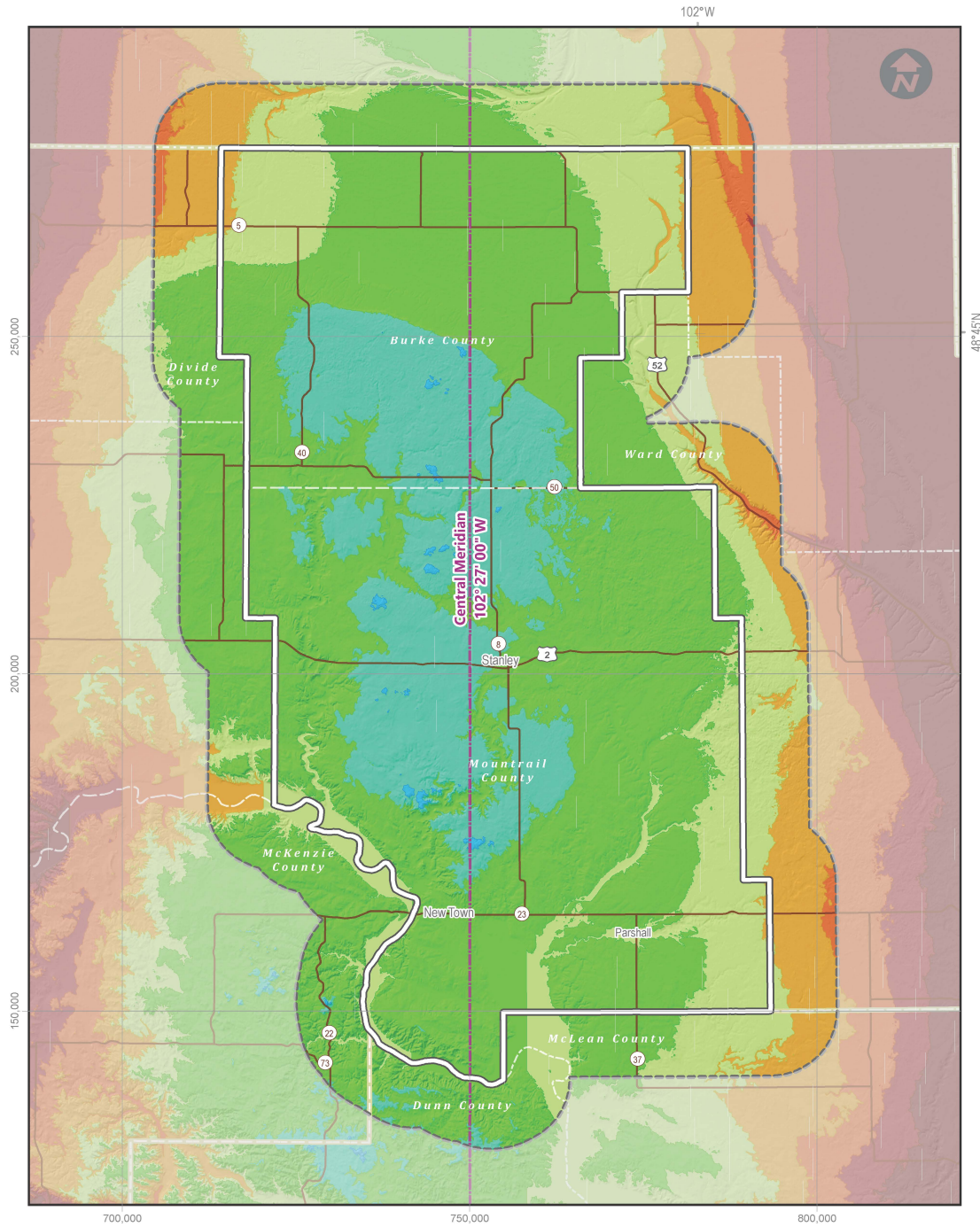
10

20

Kilometers

North Dakota

NORTH
Dakota | Transportation
Be Legendary.

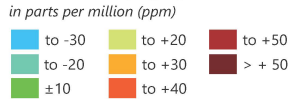


**North Dakota
Zone 2: New Town**

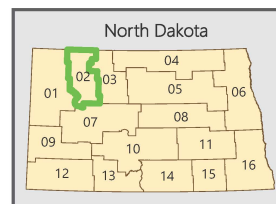
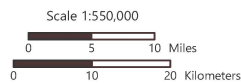
Transverse Mercator

Linear Unit: *ift*
 Latitude of Grid Origin: 46° 30' N
 Central Meridian: 102° 27' W
 False Northing: 0
 False Easting: 2,500,000
 Central Meridian Scale: 1.000094

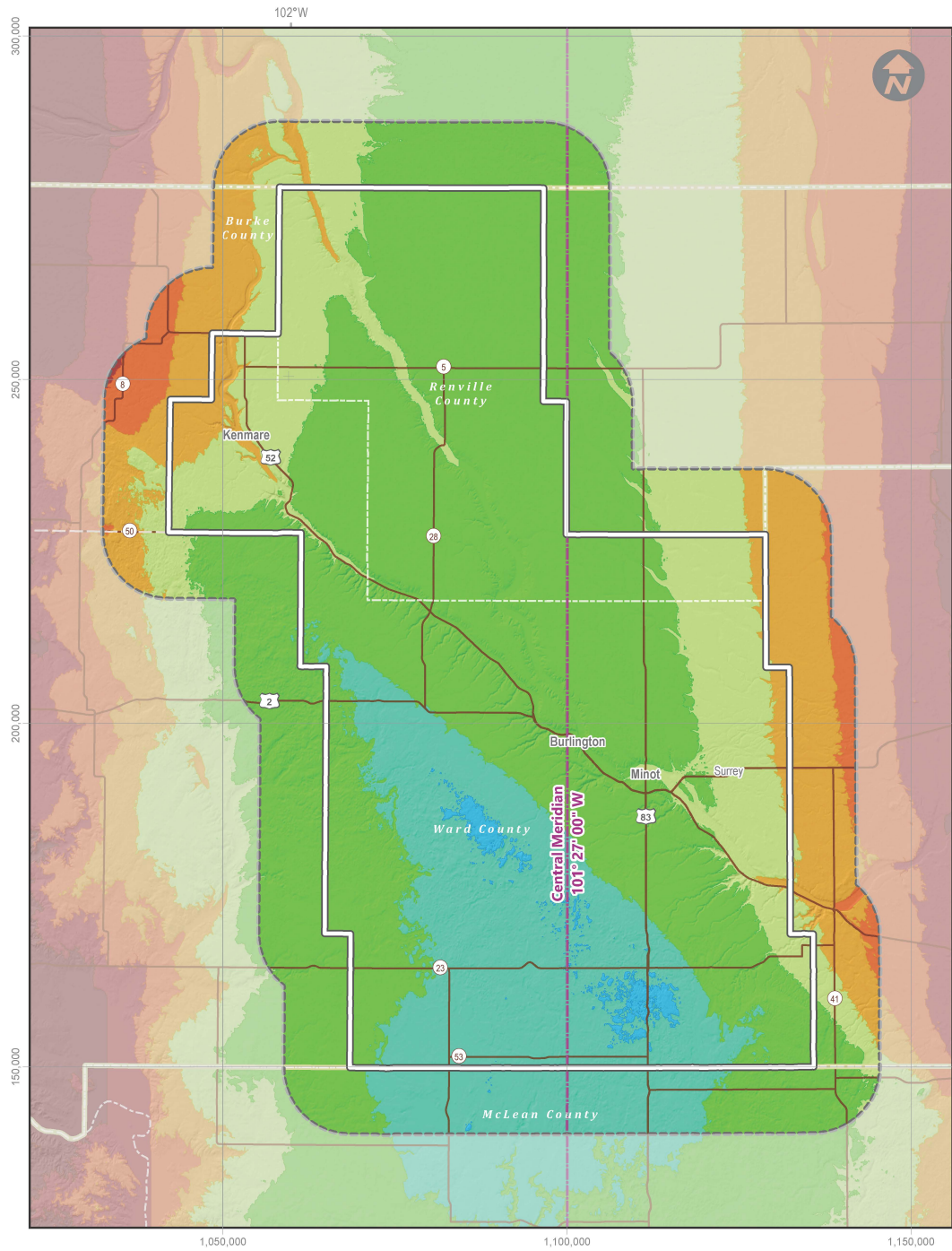
Linear Distortion at the Topographic Surface



 LDP Zone
 6-mile Buffer
 Road
 Other Zone
 County



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Dakota** | Transportation
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North Dakota Zone 3: Minot

Transverse Mercator

Linear Unit: ft
 Latitude of Grid Origin: 46° 30' N
 Central Meridian: 101° 27' W
 False Northing: 0
 False Easting: 3,625,000
 Central Meridian Scale: 1.000080

Linear Distortion at the Topographic Surface
in parts per million (ppm)

| | | | |
|--|--|--|--|
| < -50 | to -30 | to +20 | to +50 |
| to -50 | to -20 | to +30 | > +50 |
| to -40 | ±10 | to +40 | |

LDP Zone
 6-mile Buffer
 Road
 Other Zone
 County

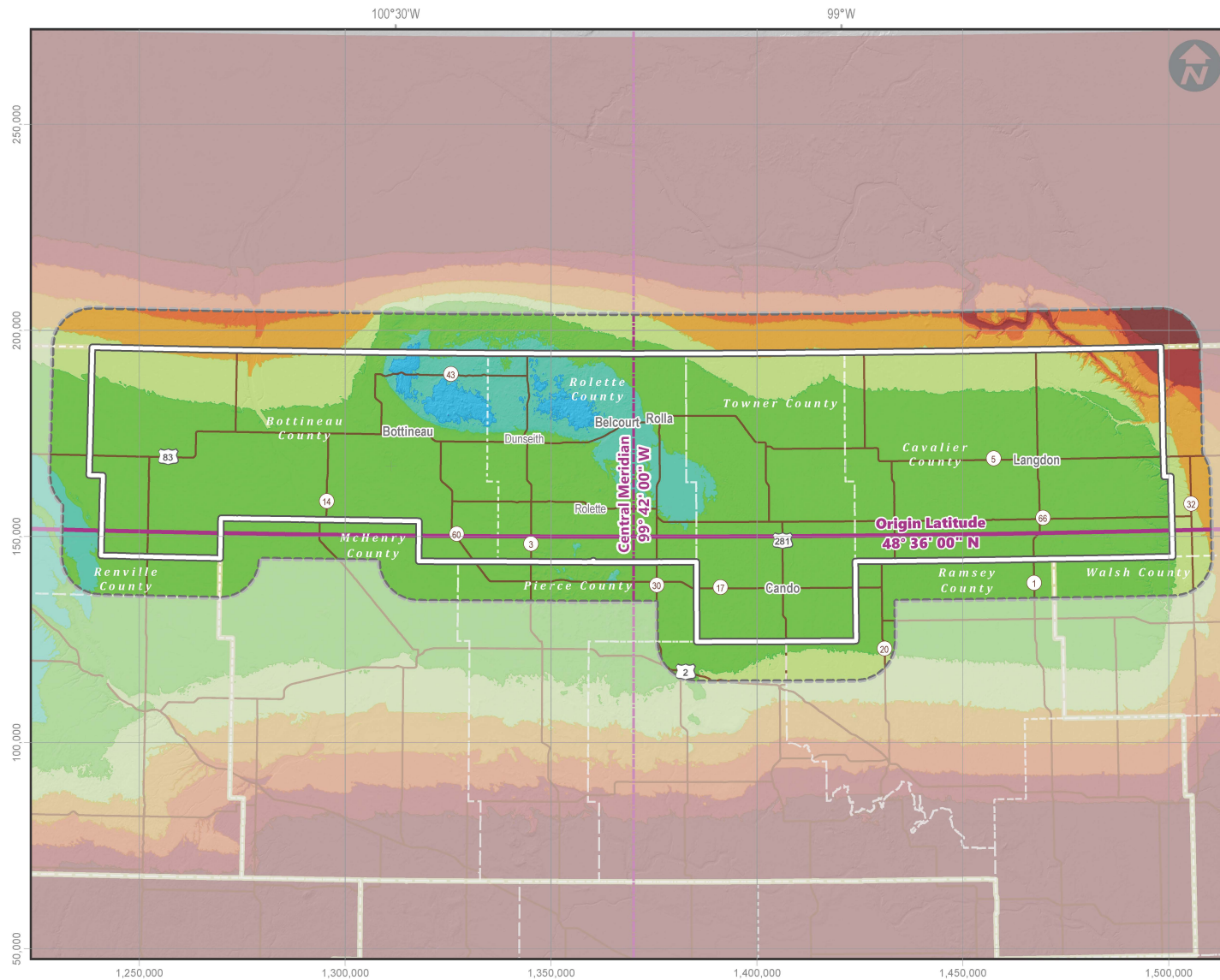
Scale 1:550,000

0 5 10 Miles

0 10 20 Kilometers

North Dakota

NORTH Dakota | Transportation
Be Legendary.



North Dakota Zone 4: Bottineau

Lambert Conformal Conic

Linear Unit: ft
 Latitude of Grid Origin: 48° 36' N
 Central Meridian: 99° 42' W
 False Northing: 500,000
 False Easting: 4,500,000
 Central Meridian Scale: 1.000066

Linear Distortion at the Topographic Surface
in parts per million (ppm)

| | | |
|--------|--------|--------|
| < -50 | to -20 | to +40 |
| to -50 | ±10 | to +50 |
| to -40 | to +20 | > + 50 |
| to -30 | to +30 | |

LDP Zone
 6-mile Buffer
 Other Zone
 County
 Road

Scale 1:925,000

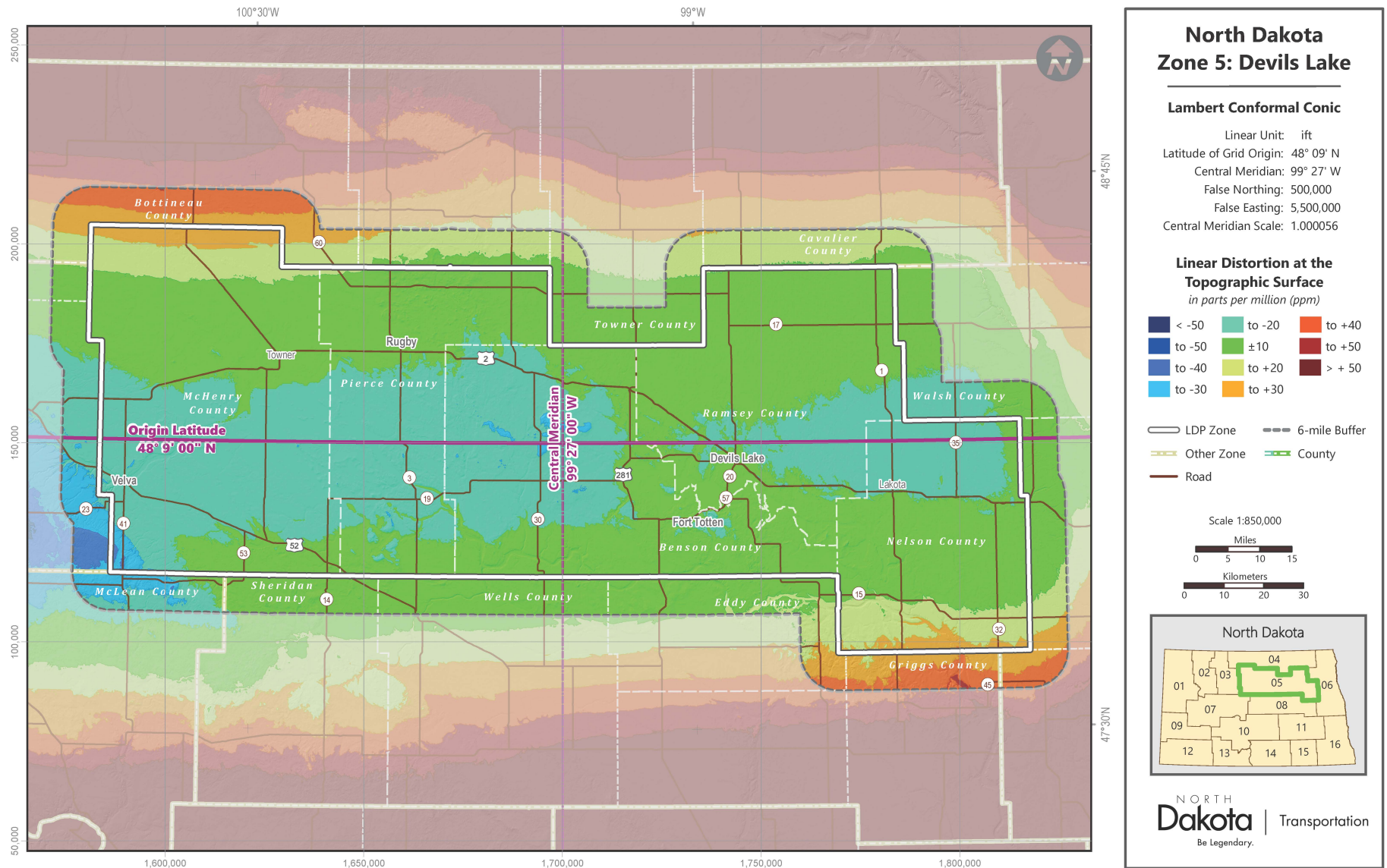
Miles

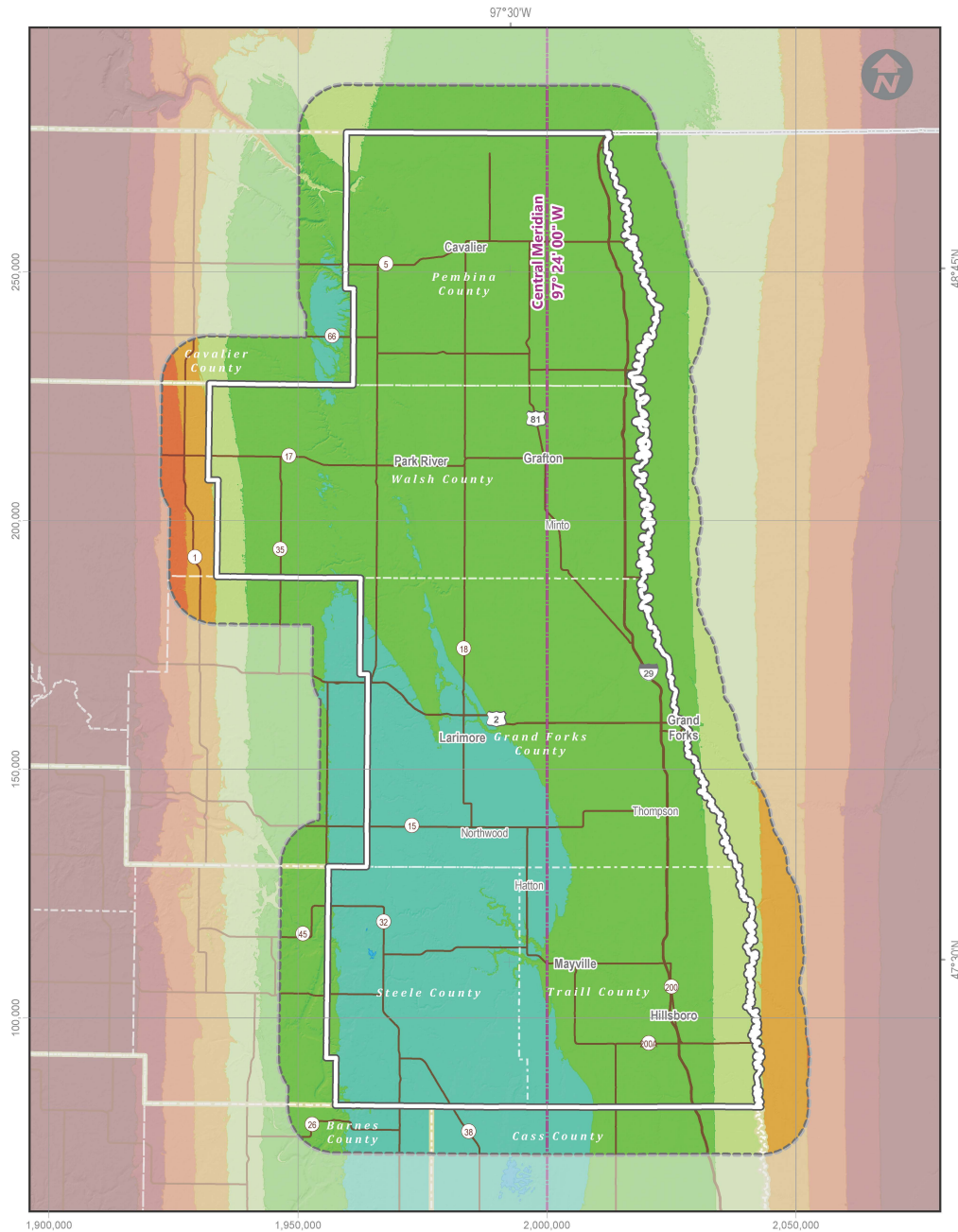
Kilometers

North Dakota

NORTH
Dakota

Transportation
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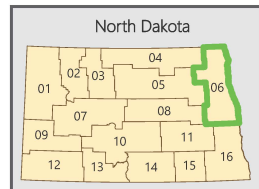
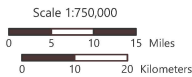
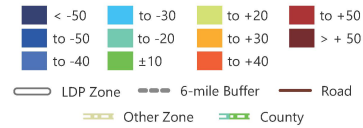
**North Dakota
Zone 6: Grand Forks**

Transverse Mercator

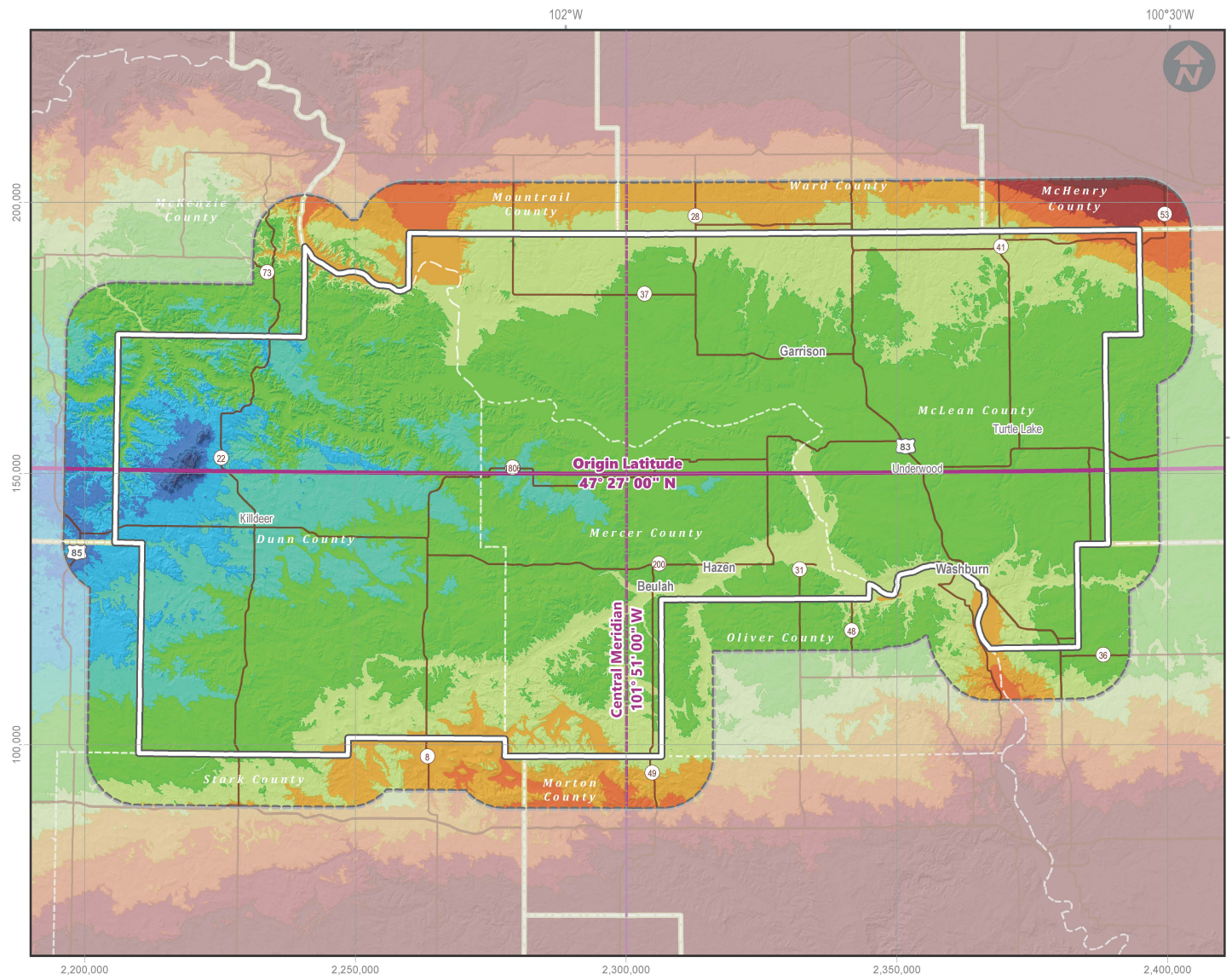
Linear Unit: *ift*
 Latitude of Grid Origin: 46° 30' N
 Central Meridian: 97° 24' W
 False Northing: 0
 False Easting: 6,500,000
 Central Meridian Scale: 1.000034

Linear Distortion at the Topographic Surface

in parts per million (ppm)



NORTH
Dakota | Transportation
 Be Legendary.



North Dakota Zone 7: Beulah

Lambert Conformal Conic

Linear Unit: ft
 Latitude of Grid Origin: 47° 27' N
 Central Meridian: 101° 51' W
 False Northing: 500,000
 False Easting: 7,500,000
 Central Meridian Scale: 1.000090

Linear Distortion at the Topographic Surface
 in parts per million (ppm)

| | | |
|-----------------------|----------------------|-------------------|
| Dark Blue: < -50 | Light Green: -20 | Red: to +40 |
| Blue: to -50 | Green: ±10 | Dark Red: to +50 |
| Light Blue: to -40 | Yellow-Green: to +20 | Dark Brown: > +50 |
| Lightest Blue: to -30 | Orange: to +30 | |

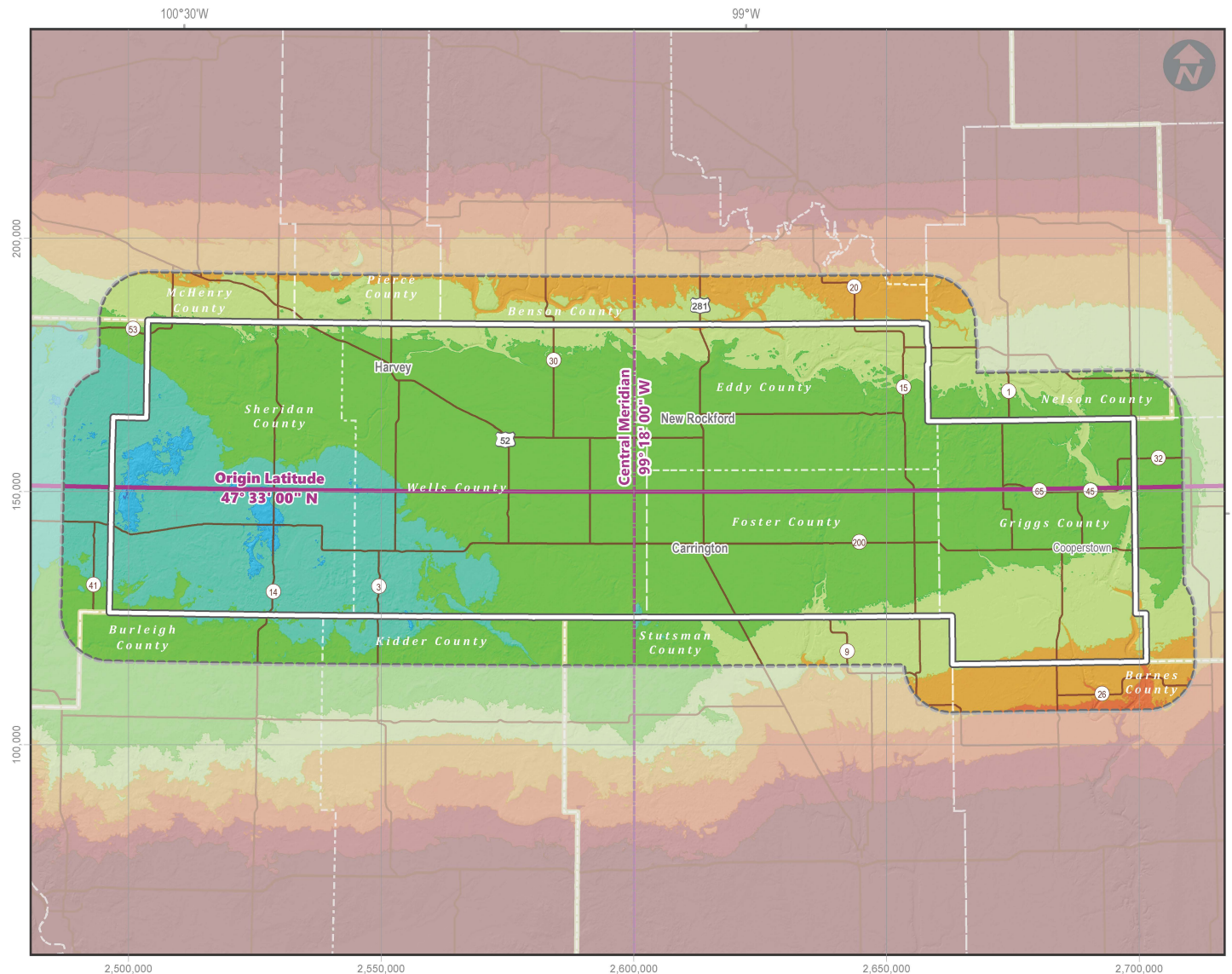
 LDP Zone 6-mile Buffer
 Other Zone County
 Road

Scale 1:700,000

Miles: 0 5 10 15
 Kilometers: 0 10 20

North Dakota

NORTH Dakota | Transportation
 Be Legendary.



North Dakota Zone 8: Carrington

Lambert Conformal Conic

Linear Unit: ft
 Latitude of Grid Origin: 47° 33' N
 Central Meridian: 99° 18' W
 False Northing: 500,000
 False Easting: 8,500,000
 Central Meridian Scale: 1.000072

**Linear Distortion at the
Topographic Surface**
in parts per million (ppm)

| | | |
|----------|----------|----------|
| ■ < -50 | ■ to -20 | ■ to +40 |
| ■ to -50 | ■ ±10 | ■ to +50 |
| ■ to -40 | ■ to +20 | ■ > +50 |
| ■ to -30 | ■ to +30 | |

 LDP Zone 6-mile Buffer
 Other Zone County
 Road

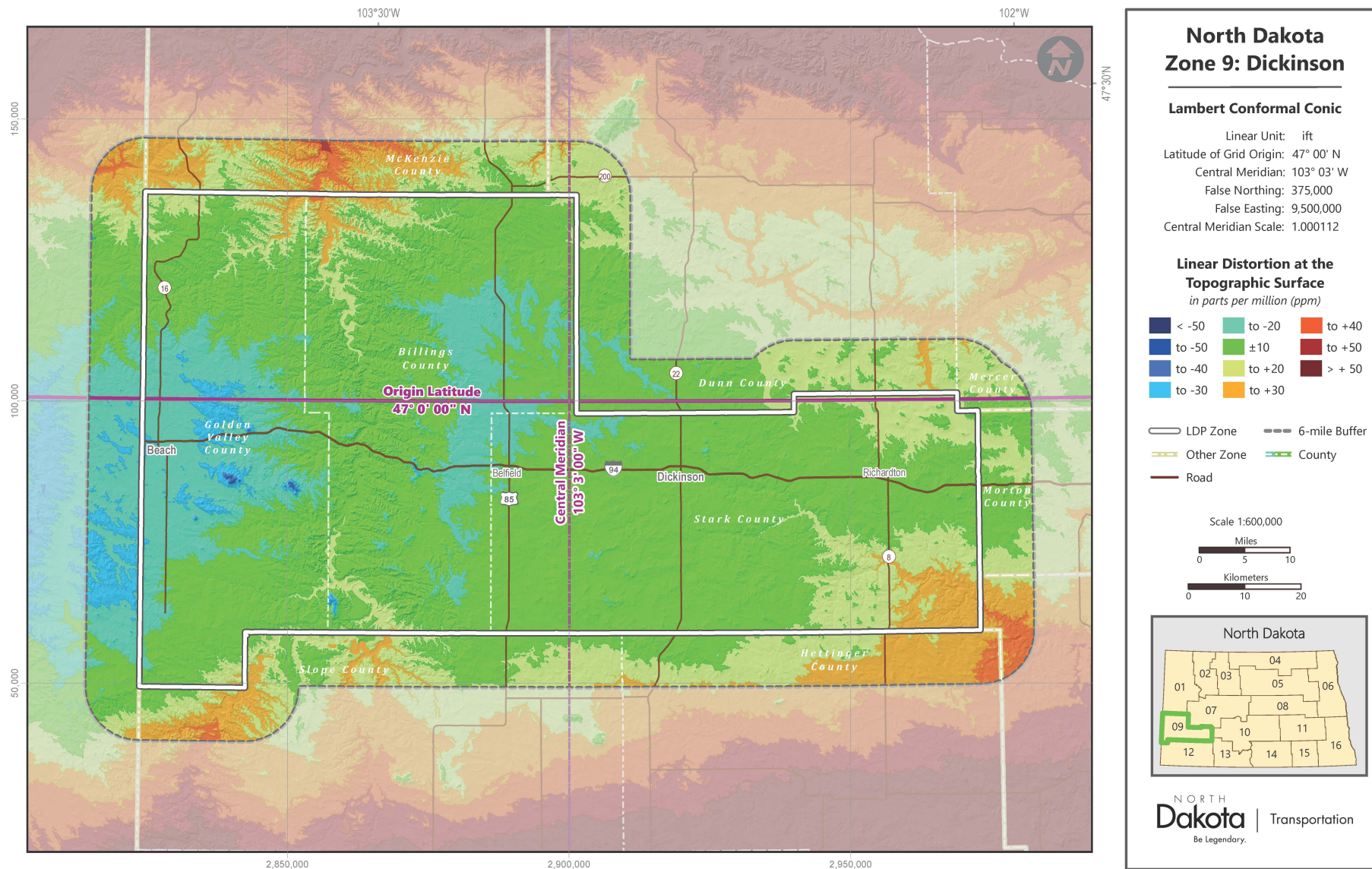
Scale 1:750,000

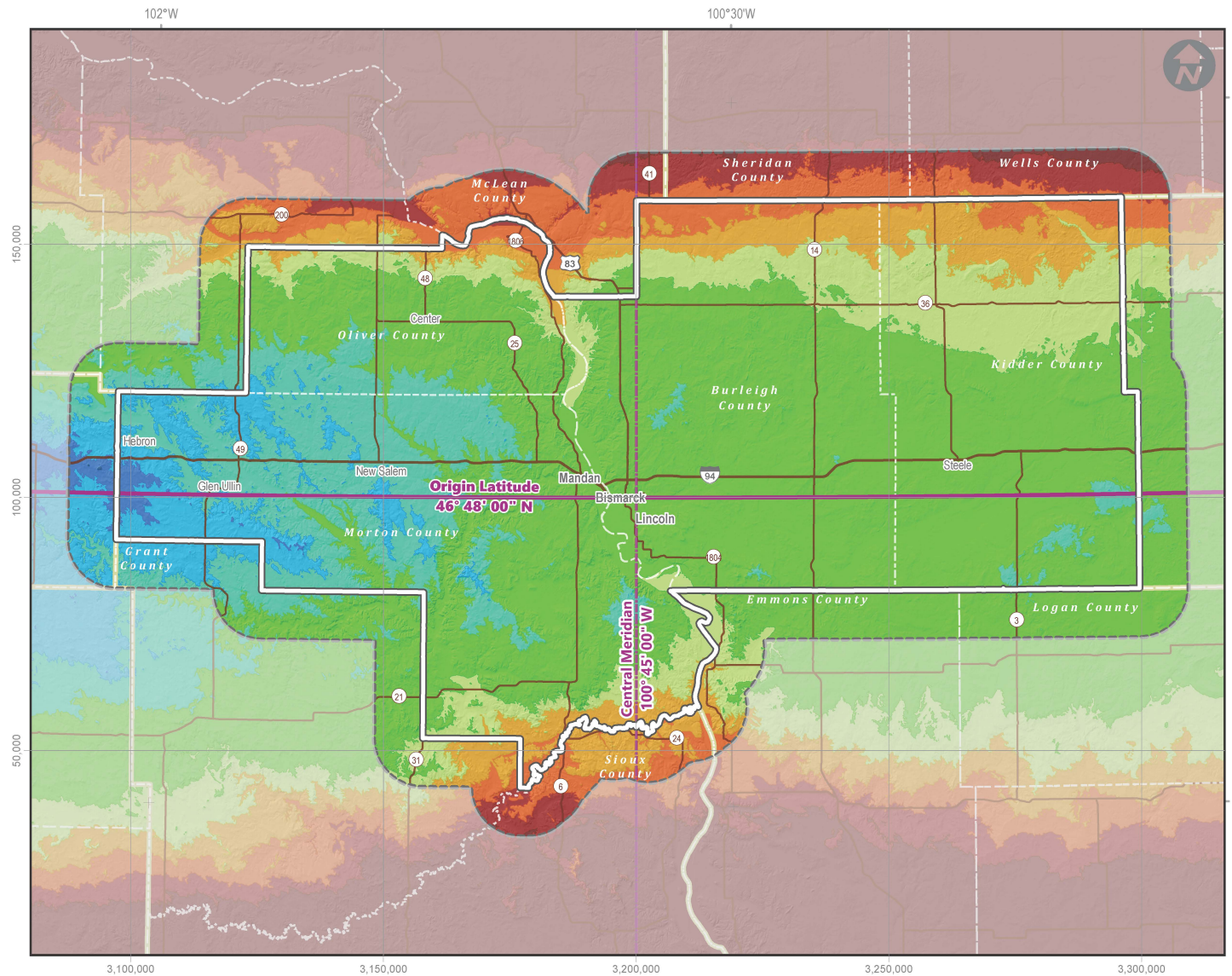
Miles

Kilometers

North Dakota

NORTH Dakota | Transportation
 Be Legendary.





North Dakota Zone10: Bismarck

Lambert Conformal Conic

Linear Unit: ft
 Latitude of Grid Origin: 46° 48' N
 Central Meridian: 100° 45' W
 False Northing: 375,000
 False Easting: 10,500,000
 Central Meridian Scale: 1.000081

**Linear Distortion at the
Topographic Surface**
in parts per million (ppm)

| | | |
|----------|----------|----------|
| ■ < -50 | ■ to -20 | ■ to +40 |
| ■ to -50 | ■ ±10 | ■ to +50 |
| ■ to -40 | ■ to +20 | ■ > +50 |
| ■ to -30 | ■ to +30 | |

 LDP Zone 6-mile Buffer
 Other Zone County
 Road

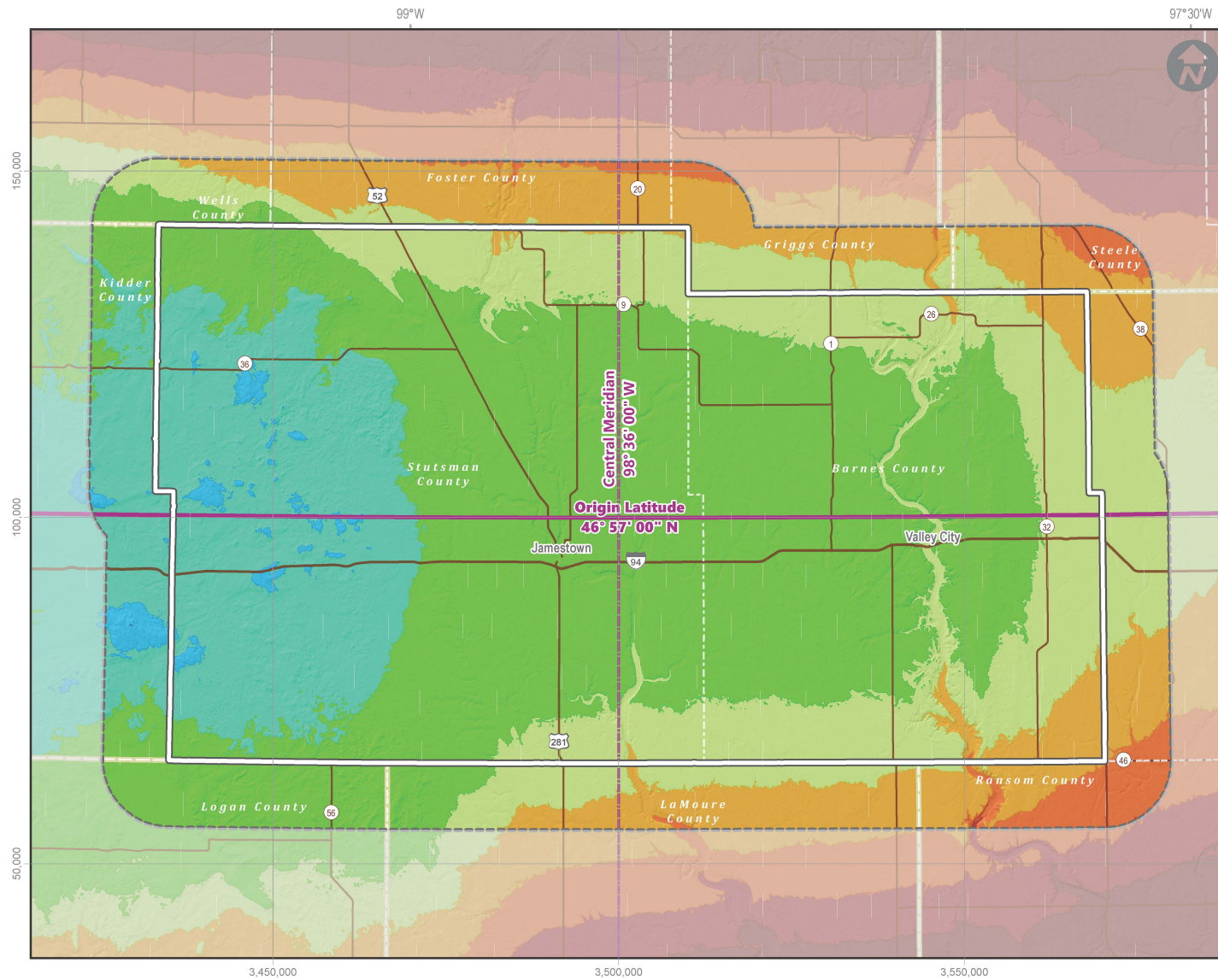
Scale 1:750,000

Miles

Kilometers

North Dakota

NORTH Dakota | Transportation
 Be Legendary.



North Dakota Zone 11: Jamestown

Lambert Conformal Conic

Linear Unit: ft
 Latitude of Grid Origin: 46° 57' N
 Central Meridian: 98° 36' W
 False Northing: 375,000
 False Easting: 11,500,000
 Central Meridian Scale: 1.000067

**Linear Distortion at the
Topographic Surface**
in parts per million (ppm)

| | | |
|--------|--------|--------|
| < -50 | to -20 | to +40 |
| to -50 | ±10 | to +50 |
| to -40 | to +20 | > +50 |
| to -30 | to +30 | |

LDP Zone
 6-mile Buffer
 Other Zone
 County
 Road

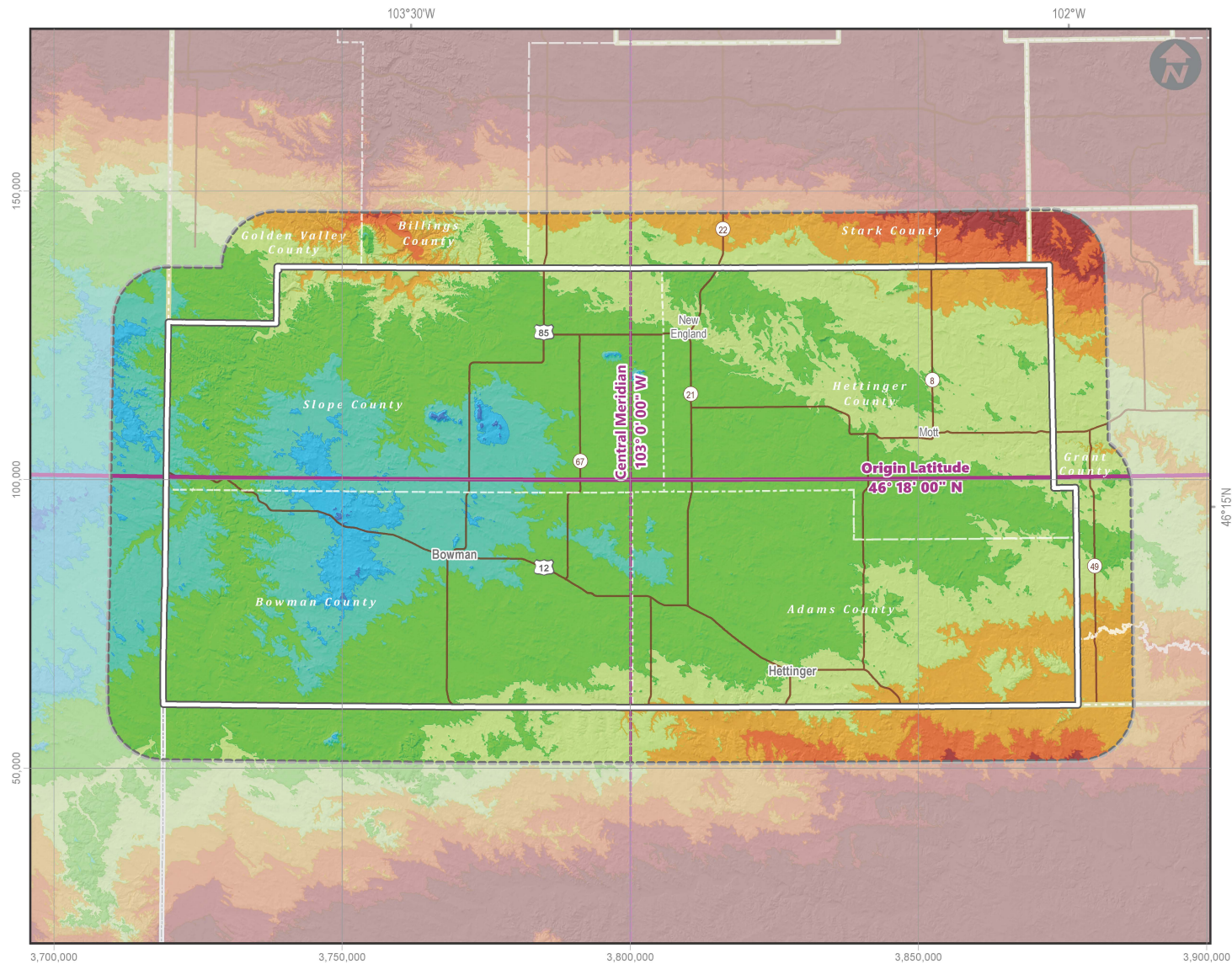
Scale 1:550,000

Miles

Kilometers

North Dakota

NORTH Dakota | Transportation
 Be Legendary.



North Dakota Zone 12: Bowman

Lambert Conformal Conic

Linear Unit: ft
 Latitude of Grid Origin: 46° 18' N
 Central Meridian: 103° 00' W
 False Northing: 375000
 False Easting: 12,500,000
 Central Meridian Scale: 1.000124

Linear Distortion at the Topographic Surface
in parts per million (ppm)

| | | |
|-------------|------------|-----------|
| Dark Blue | Light Blue | Red |
| < -50 | -20 | to +40 |
| Blue | Green | Dark Red |
| to -50 | ±10 | to +50 |
| Light Blue | Yellow | Dark Blue |
| to -40 | to +20 | > +50 |
| Light Green | Orange | |
| to -30 | to +30 | |

LDP Zone
 6-mile Buffer
 Other Zone
 County
 Road

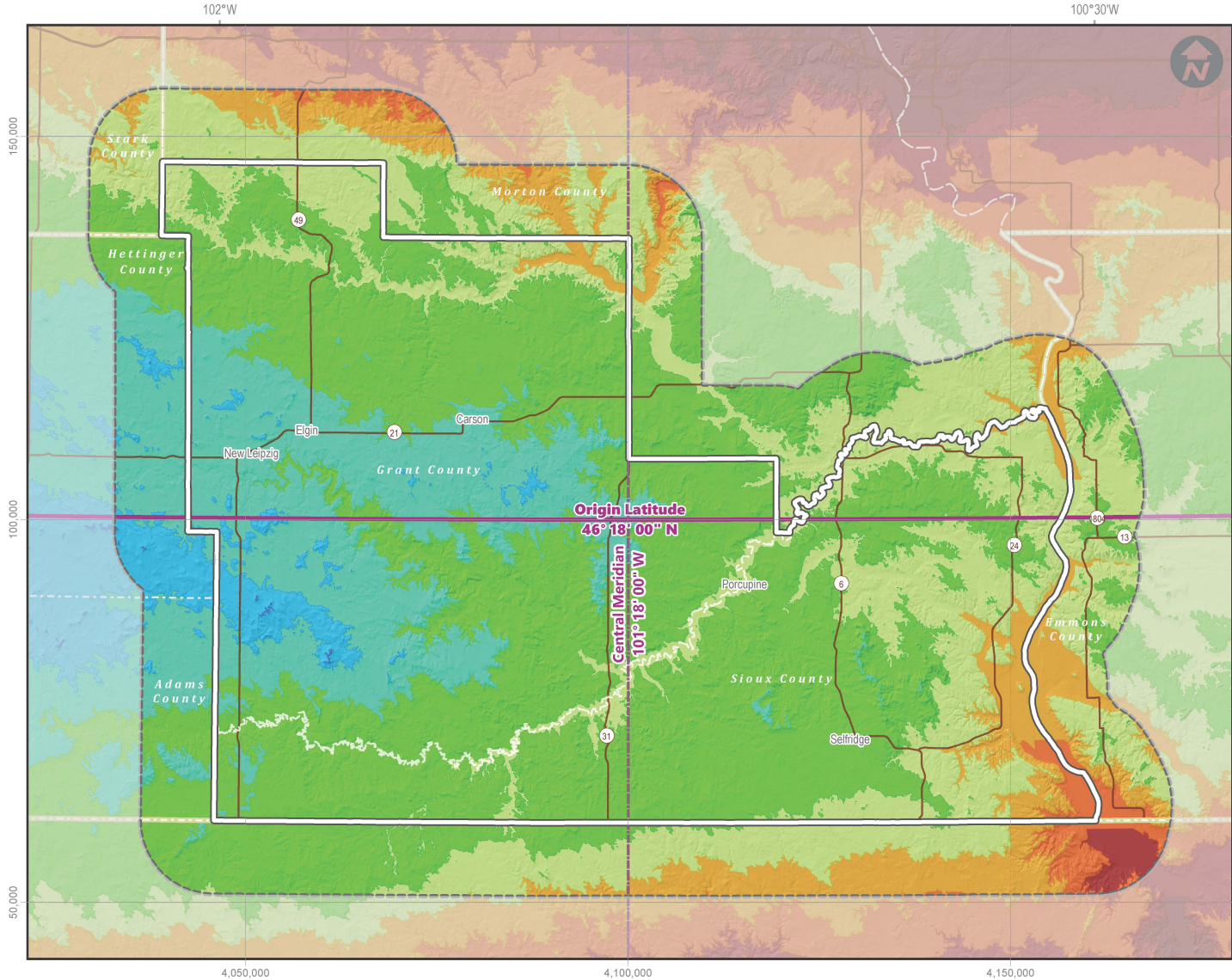
Scale 1:650,000

Miles
 0 5 10

Kilometers
 0 10 20

North Dakota

NORTH Dakota | Transportation
 Be Legendary.



North Dakota Zone 13: Cannon Ball

Lambert Conformal Conic

- Linear Unit: ft
- Latitude of Grid Origin: 46° 18' N
- Central Meridian: 101° 18' W
- False Northing: 375,000
- False Easting: 413,500,000
- Central Meridian Scale: 1.000093

Linear Distortion at the Topographic Surface
in parts per million (ppm)

| | | |
|--------|--------|--------|
| < -50 | to -20 | to +40 |
| to -50 | ±10 | to +50 |
| to -40 | to +20 | > +50 |
| to -30 | to +30 | |

- LDP Zone
- 6-mile Buffer
- Other Zone
- County
- Road

Scale 1:500,000

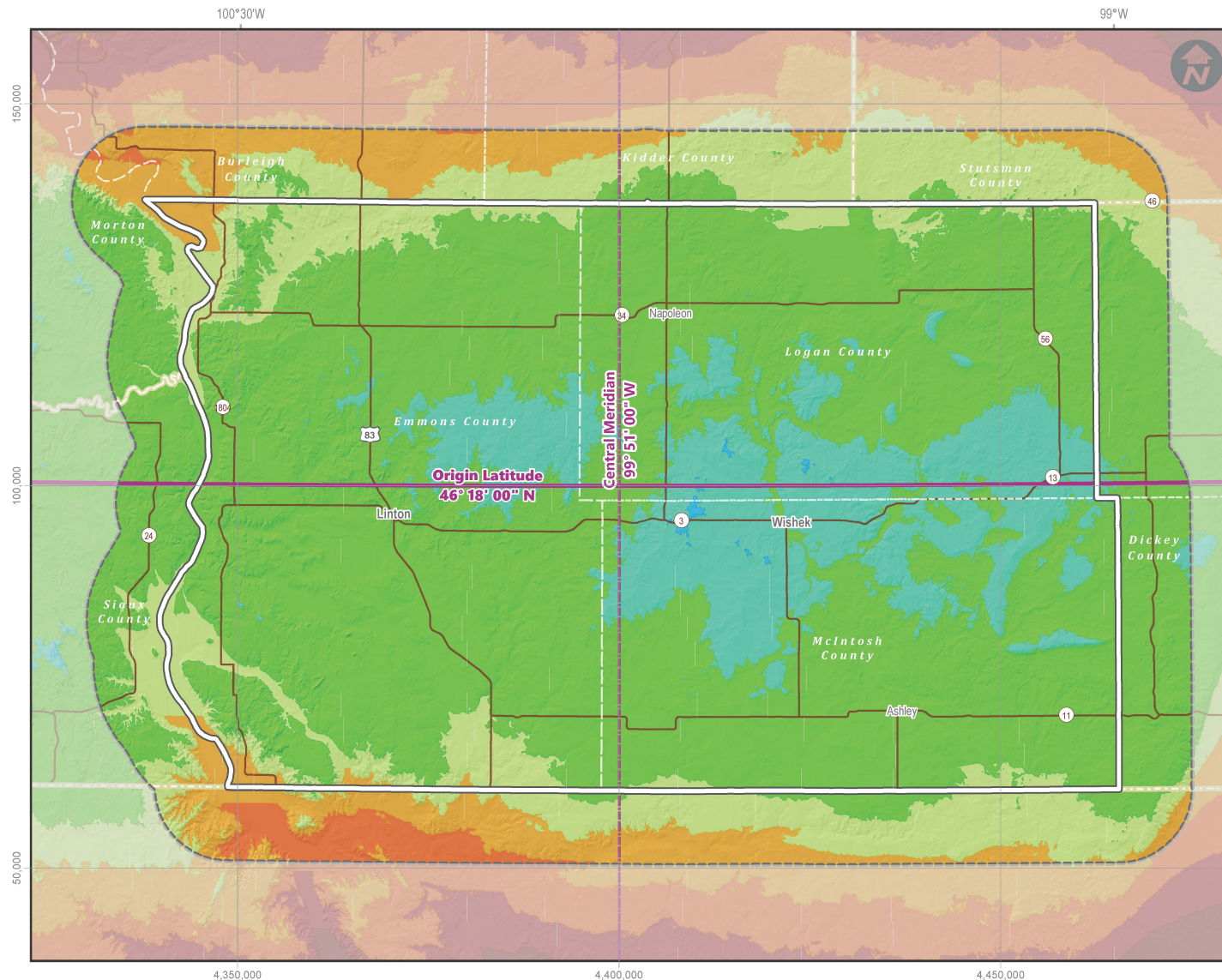
Miles

Kilometers

North Dakota

NORTH
Dakota

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North Dakota Zone 14: Linton

Lambert Conformal Conic

Linear Unit: ft
 Latitude of Grid Origin: 46° 18' N
 Central Meridian: 99° 51' W
 False Northing: 375,000
 False Easting: 14,375,000
 Central Meridian Scale: 1.000082

Linear Distortion at the Topographic Surface
 in parts per million (ppm)

| | | |
|------------|-------------|----------|
| Dark Blue | Light Green | Red |
| < -50 | to -20 | to +40 |
| Blue | Green | Dark Red |
| to -50 | ±10 | to +50 |
| Dark Blue | Yellow | Dark Red |
| to -40 | to +20 | > +50 |
| Light Blue | Orange | |
| to -30 | to +30 | |

 LDP Zone 6-mile Buffer
 Other Zone County
 Road

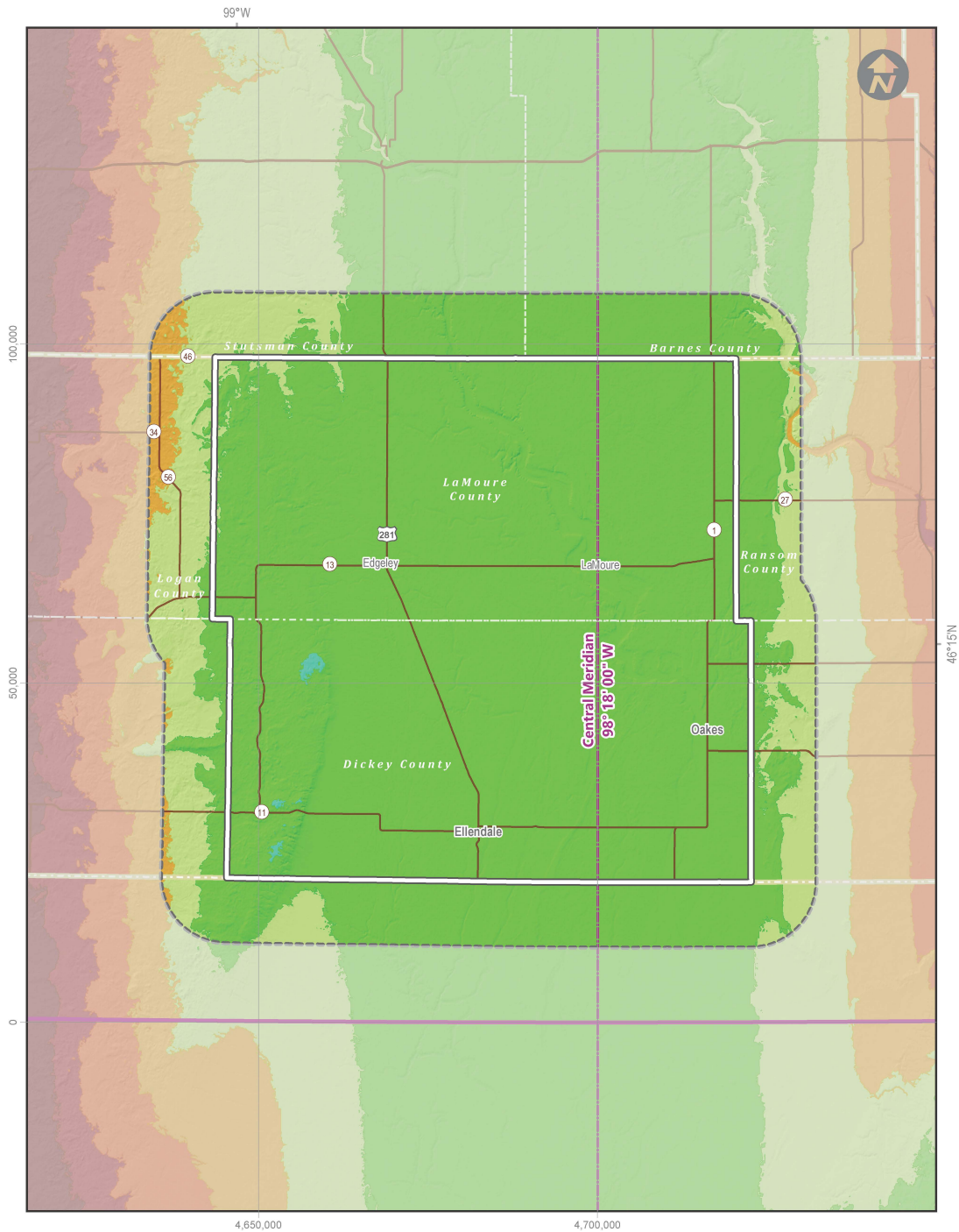
Scale 1:500,000

Miles
0 5 10

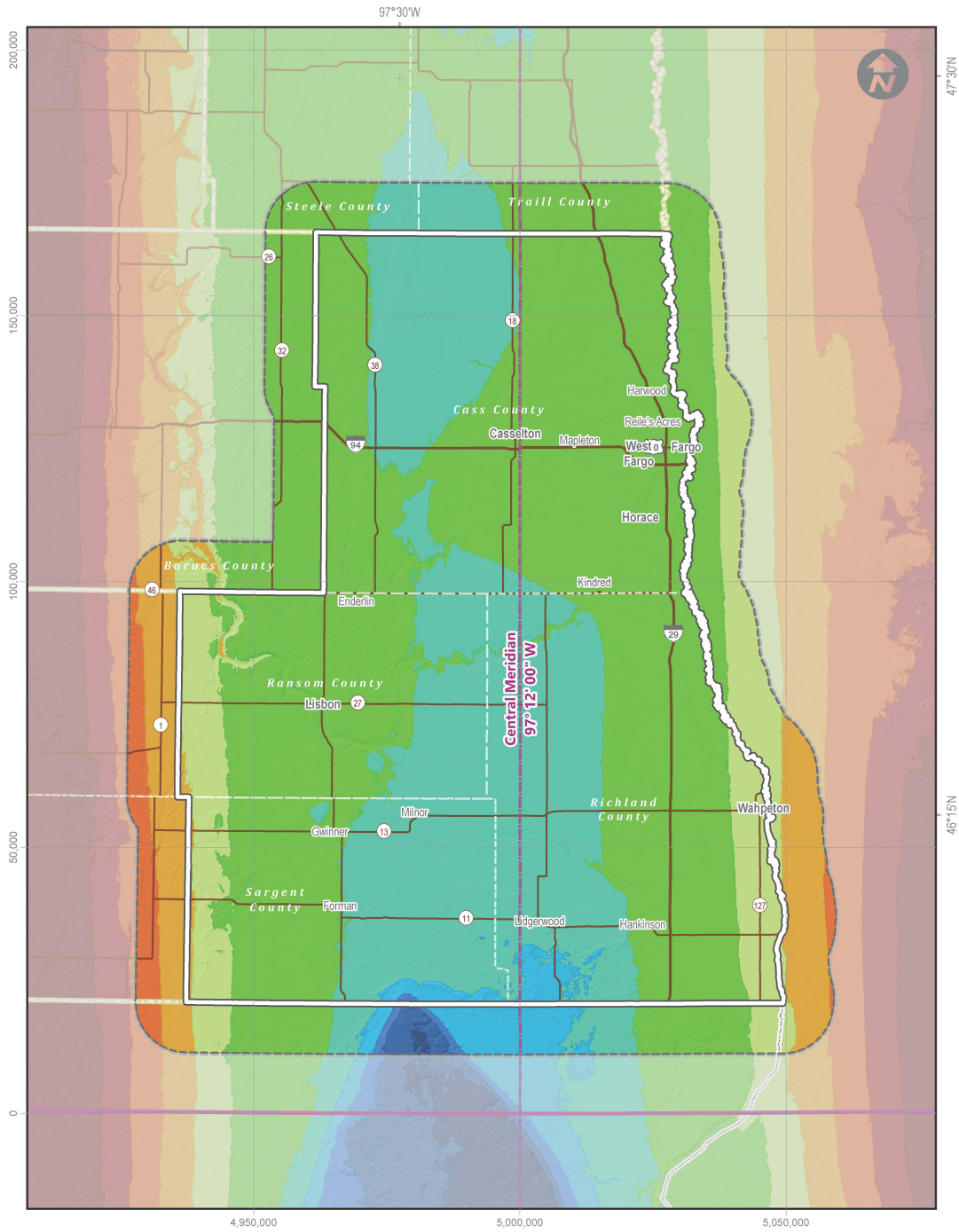
Kilometers
0 10

North Dakota

NORTH Dakota | Transportation
 Be Legendary.



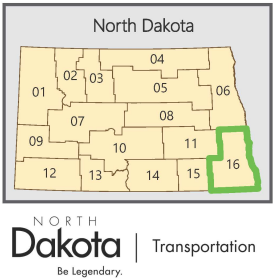
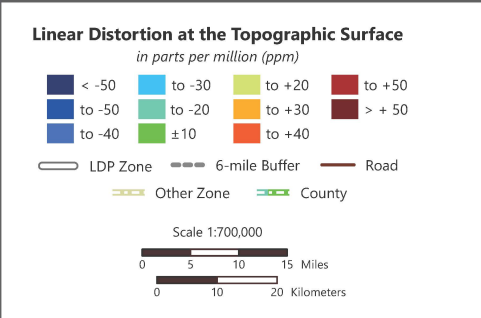
| | | | | | | | | | | | | | | |
|---|---|--------|--------|--------|--------|--------|--------|--------|-------|--------|-----|--------|--|---|
| <p>North Dakota Zone 15: Oakes</p> <p>Transverse Mercator</p> <p>Linear Unit: ft Latitude of Grid Origin: 45° 45' N Central Meridian: 98° 18' W False Northing: 0 False Easting: 15,375,000 Central Meridian Scale: 1.000061</p> | <p>Linear Distortion at the Topographic Surface <i>in parts per million (ppm)</i></p> <table border="0"> <tr> <td> < -50</td> <td> to -30</td> <td> to +20</td> <td> to +50</td> </tr> <tr> <td> to -50</td> <td> to -20</td> <td> to +30</td> <td> > +50</td> </tr> <tr> <td> to -40</td> <td> ±10</td> <td> to +40</td> <td></td> </tr> </table> <p> LDP Zone 6-mile Buffer Road Other Zone County </p> <p>Scale 1:550,000</p> <p>0 5 10 Miles 0 10 20 Kilometers</p> | < -50 | to -30 | to +20 | to +50 | to -50 | to -20 | to +30 | > +50 | to -40 | ±10 | to +40 | | <p>North Dakota</p> <p>NORTH Dakota Transportation Be Legendary.</p> |
| < -50 | to -30 | to +20 | to +50 | | | | | | | | | | | |
| to -50 | to -20 | to +30 | > +50 | | | | | | | | | | | |
| to -40 | ±10 | to +40 | | | | | | | | | | | | |



**North Dakota
Zone 16: Fargo**

Transverse Mercator

Linear Unit: ft
 Latitude of Grid Origin: 45° 45' N
 Central Meridian: 97° 12' W
 False Northing: 0
 False Easting: 16,375,000
 Central Meridian Scale: 1.000032



Appendix B. Distortion and ellipsoid height values for incorporated places

Table B1. Distortion and ellipsoid height values for select incorporated places

There are 357 incorporated places in North Dakota. This table represents the 94 places that have a 2019 estimated population of 500 or more. The locations represent the approximate centroid based on the 2018 boundaries according to the U.S. Census Bureau. Ellipsoid heights have been rounded to the nearest meter and foot.

Population data source: U.S. Census Bureau population 2019 estimates

| Name | 2019 Population Estimate | Latitude | Longitude | Ellipsoid height (m) | Ellipsoid height (ift) | Zone | Zone Name | Linear distortion (ppm) |
|-------------|--------------------------|----------|-----------|----------------------|------------------------|------|-------------|-------------------------|
| Ashley | 661 | 46.0341 | -99.3715 | 596 | 1,954 | 14 | Linton | -0.6539 |
| Beach | 1,043 | 46.9181 | -104.0044 | 834 | 2,735 | 9 | Dickinson | -17.6345 |
| Belfield | 1,012 | 46.8853 | -103.1996 | 787 | 2,583 | 9 | Dickinson | -9.4231 |
| Beulah | 3,139 | 47.2633 | -101.7779 | 571 | 1,873 | 7 | Beulah | 5.7916 |
| Bismarck | 73,529 | 46.8083 | -100.7837 | 499 | 1,636 | 10 | Bismarck | 2.8449 |
| Bottineau | 2,154 | 48.8272 | -100.4457 | 491 | 1,611 | 4 | Bottineau | -3.1152 |
| Bowman | 1,599 | 46.1831 | -103.3949 | 885 | 2,903 | 12 | Bowman | -12.6377 |
| Burlington | 1,201 | 48.2753 | -101.4288 | 508 | 1,668 | 3 | Minot | 0.3511 |
| Cando | 1,057 | 48.4867 | -99.2099 | 426 | 1,398 | 4 | Bottineau | 1.1636 |
| Carrington | 1,980 | 47.4497 | -99.1262 | 455 | 1,494 | 8 | Carrington | 2.1607 |
| Casselton | 2,475 | 46.9005 | -97.2112 | 258 | 845 | 16 | Fargo | -8.3680 |
| Cavalier | 1,238 | 48.7939 | -97.6223 | 242 | 795 | 6 | Grand Forks | -0.6941 |
| Center | 578 | 47.1164 | -101.2996 | 615 | 2,017 | 10 | Bismarck | -0.1270 |
| Cooperstown | 903 | 47.4444 | -98.1240 | 411 | 1,348 | 8 | Carrington | 9.2698 |
| Crosby | 1,285 | 48.9142 | -103.2949 | 581 | 1,906 | 1 | Williston | 2.5355 |
| Devils Lake | 7,320 | 48.1145 | -98.8585 | 420 | 1,377 | 5 | Devils Lake | -9.5945 |
| Dickinson | 23,133 | 46.8792 | -102.7896 | 741 | 2,431 | 9 | Dickinson | -1.9421 |
| Drayton | 747 | 48.5711 | -97.1778 | 216 | 707 | 6 | Grand Forks | 3.5226 |
| Dunseith | 764 | 48.8131 | -100.0610 | 515 | 1,689 | 4 | Bottineau | -7.7563 |
| Edgeley | 556 | 46.3591 | -98.7157 | 454 | 1,491 | 15 | Oakes | 2.3521 |
| Elgin | 603 | 46.4039 | -101.8460 | 686 | 2,251 | 13 | Cannon Ball | -12.9057 |
| Ilendale | 1,246 | 46.0028 | -98.5270 | 416 | 1,365 | 15 | Oakes | -0.4229 |
| Enderlin | 838 | 46.6230 | -97.6015 | 316 | 1,036 | 16 | Fargo | -5.8955 |
| Fargo | 124,662 | 46.8772 | -96.7898 | 246 | 807 | 16 | Fargo | 5.4582 |
| Forman | 504 | 46.1077 | -97.6365 | 357 | 1,172 | 16 | Fargo | -9.9895 |
| Garrison | 1,462 | 47.6522 | -101.4157 | 565 | 1,853 | 7 | Beulah | 7.6965 |
| Glen Ullin | 717 | 46.8150 | -101.8299 | 634 | 2,080 | 10 | Bismarck | -18.3606 |
| Grafton | 4,157 | 48.4122 | -97.4106 | 225 | 738 | 6 | Grand Forks | -1.2230 |
| Grand Forks | 55,839 | 47.9253 | -97.0329 | 225 | 738 | 6 | Grand Forks | 8.0118 |
| Gwinner | 937 | 46.2258 | -97.6626 | 363 | 1,190 | 16 | Fargo | -9.1947 |
| Hankinson | 885 | 46.0697 | -96.9017 | 297 | 974 | 16 | Fargo | -8.0143 |
| Harvey | 1,646 | 47.7697 | -99.9354 | 465 | 1,527 | 8 | Carrington | 6.3909 |
| Harwood | 829 | 46.9794 | -96.8806 | 243 | 799 | 16 | Fargo | 1.0922 |

North Dakota Coordinate Reference System

| Name | 2019 Population Estimate | Latitude | Longitude | Ellipsoid height (m) | Ellipsoid height (ift) | Zone | Zone Name | Linear distortion (ppm) |
|---------------|--------------------------|----------|-----------|----------------------|------------------------|------|-------------|-------------------------|
| Hatton | 743 | 47.6397 | -97.4534 | 298 | 978 | 6 | Grand Forks | -12.5297 |
| Hazen | 2,311 | 47.2944 | -101.6227 | 546 | 1,790 | 7 | Beulah | 8.1344 |
| Hebron | 675 | 46.9006 | -102.0454 | 663 | 2,176 | 10 | Bismarck | -21.4186 |
| Hettinger | 1,155 | 46.0014 | -102.6368 | 813 | 2,667 | 12 | Bowman | 10.0973 |
| Hillsboro | 1,624 | 47.4039 | -97.0620 | 249 | 817 | 6 | Grand Forks | 2.9627 |
| Horace | 2,944 | 46.7589 | -96.9037 | 250 | 820 | 16 | Fargo | -0.8840 |
| Jamestown | 15,084 | 46.9105 | -98.7084 | 426 | 1,398 | 11 | Jamestown | 0.4450 |
| Kenmare | 1,021 | 48.6747 | -102.0827 | 570 | 1,871 | 3 | Minot | 17.3064 |
| Killdeer | 1,144 | 47.3720 | -102.7541 | 681 | 2,236 | 7 | Beulah | -15.8912 |
| Kindred | 781 | 46.6486 | -97.0170 | 260 | 852 | 16 | Fargo | -6.2978 |
| Lakota | 625 | 48.0428 | -98.3362 | 435 | 1,427 | 5 | Devils Lake | -10.4225 |
| LaMoure | 883 | 46.3572 | -98.2945 | 394 | 1,293 | 15 | Oakes | -0.7603 |
| Langdon | 1,752 | 48.7600 | -98.3682 | 468 | 1,535 | 4 | Bottineau | -3.4183 |
| Larimore | 1,288 | 47.9067 | -97.6268 | 314 | 1,030 | 6 | Grand Forks | -11.6515 |
| Lidgerwood | 616 | 46.0755 | -97.1518 | 316 | 1,036 | 16 | Fargo | -17.3304 |
| Lincoln | 3,817 | 46.7625 | -100.7004 | 505 | 1,658 | 10 | Bismarck | 2.0160 |
| Linton | 978 | 46.2667 | -100.2329 | 539 | 1,769 | 14 | Linton | -2.3464 |
| Lisbon | 2,051 | 46.4416 | -97.6812 | 335 | 1,100 | 16 | Fargo | -3.7591 |
| Mandan | 22,752 | 46.8267 | -100.8896 | 511 | 1,678 | 10 | Bismarck | 0.9479 |
| Mapleton | 1,238 | 46.8891 | -97.0526 | 249 | 816 | 16 | Fargo | -5.4383 |
| Mayville | 1,802 | 47.4980 | -97.3245 | 270 | 886 | 6 | Grand Forks | -7.9303 |
| Milnor | 634 | 46.2591 | -97.4562 | 314 | 1,032 | 16 | Fargo | -12.4968 |
| Minot | 47,382 | 48.2325 | -101.2963 | 486 | 1,595 | 3 | Minot | 5.4227 |
| Minto | 592 | 48.2917 | -97.3715 | 223 | 731 | 6 | Grand Forks | -0.8868 |
| Mohall | 723 | 48.7634 | -101.5132 | 476 | 1,561 | 3 | Minot | 5.7182 |
| Mott | 724 | 46.3725 | -102.3271 | 734 | 2,409 | 12 | Bowman | 9.6964 |
| Napoleon | 752 | 46.5034 | -99.7620 | 589 | 1,931 | 14 | Linton | -3.9696 |
| New England | 600 | 46.5392 | -102.8682 | 794 | 2,605 | 12 | Bowman | 8.2090 |
| New Rockford | 1,339 | 47.6800 | -99.1379 | 445 | 1,459 | 8 | Carrington | 4.8615 |
| New Salem | 989 | 46.8450 | -101.4113 | 632 | 2,073 | 10 | Bismarck | -17.7196 |
| New Town | 2,592 | 47.9808 | -102.4902 | 582 | 1,909 | 2 | New Town | 2.9263 |
| Northwood | 895 | 47.7342 | -97.5668 | 311 | 1,020 | 6 | Grand Forks | -12.7942 |
| Oakes | 1,701 | 46.1386 | -98.0904 | 375 | 1,230 | 15 | Oakes | 5.4357 |
| Park River | 1,338 | 48.3986 | -97.7412 | 283 | 929 | 6 | Grand Forks | -2.5204 |
| Parshall | 1,288 | 47.9533 | -102.1349 | 605 | 1,985 | 2 | New Town | 6.0153 |
| Pembina | 544 | 48.9664 | -97.2437 | 213 | 697 | 6 | Grand Forks | 2.2977 |
| Portland | 595 | 47.4983 | -97.3704 | 276 | 906 | 6 | Grand Forks | -9.2114 |
| Ray | 901 | 48.3445 | -103.1652 | 686 | 2,251 | 1 | Williston | -10.0376 |
| Reile's Acres | 700 | 46.9269 | -96.8665 | 245 | 804 | 16 | Fargo | 1.5295 |
| Richardton | 561 | 46.8839 | -102.3157 | 705 | 2,312 | 9 | Dickinson | 3.5645 |
| Rolette | 593 | 48.6608 | -99.8415 | 472 | 1,549 | 4 | Bottineau | -7.4430 |
| Rolla | 1,252 | 48.8578 | -99.6179 | 536 | 1,759 | 4 | Bottineau | -7.9114 |
| Rugby | 2,590 | 48.3689 | -99.9962 | 450 | 1,477 | 5 | Devils Lake | -7.2658 |

| Name | 2019 Population Estimate | Latitude | Longitude | Ellipsoid height (m) | Ellipsoid height (ift) | Zone | Zone Name | Linear distortion (ppm) |
|--------------|--------------------------|----------|-----------|----------------------|------------------------|------|-------------|-------------------------|
| Stanley | 2,677 | 48.3172 | -102.3905 | 668 | 2,192 | 2 | New Town | -10.4639 |
| Steele | 711 | 46.8547 | -99.9159 | 532 | 1,745 | 10 | Bismarck | -1.9277 |
| Surrey | 1,393 | 48.2364 | -101.1335 | 469 | 1,539 | 3 | Minot | 13.2873 |
| Thompson | 1,021 | 47.7736 | -97.1098 | 237 | 778 | 6 | Grand Forks | 2.6203 |
| Tioga | 1,339 | 48.3972 | -102.9382 | 683 | 2,241 | 1 | Williston | 2.5962 |
| Towner | 526 | 48.3458 | -100.4054 | 428 | 1,403 | 5 | Devils Lake | -5.1860 |
| Turtle Lake | 554 | 47.5200 | -100.8901 | 551 | 1,807 | 7 | Beulah | 4.3937 |
| Underwood | 735 | 47.4564 | -101.1371 | 588 | 1,929 | 7 | Beulah | -2.1475 |
| Valley City | 6,323 | 46.9233 | -98.0032 | 386 | 1,265 | 11 | Jamestown | 6.6612 |
| Velva | 1,190 | 48.0561 | -100.9293 | 468 | 1,534 | 5 | Devils Lake | -15.9423 |
| Wahpeton | 7,734 | 46.2652 | -96.6059 | 265 | 870 | 16 | Fargo | 16.2237 |
| Walhalla | 907 | 48.9233 | -97.9181 | 294 | 963 | 6 | Grand Forks | 5.7092 |
| Washburn | 1,264 | 47.2892 | -101.0290 | 531 | 1,742 | 7 | Beulah | 10.6750 |
| Watford City | 7,835 | 47.8022 | -103.2832 | 630 | 2,067 | 1 | Williston | -4.8258 |
| West Fargo | 37,058 | 46.8750 | -96.9004 | 246 | 808 | 16 | Fargo | -0.2162 |
| Williston | 29,033 | 48.1470 | -103.6180 | 560 | 1,837 | 1 | Williston | 6.1504 |
| Wilton | 716 | 47.1586 | -100.7835 | 607 | 1,991 | 7 | Beulah | 7.7508 |
| Wishek | 891 | 46.2569 | -99.5571 | 610 | 2,002 | 14 | Linton | -13.3698 |

Appendix C. Equations for computing projection grid point scale factors

The projection grid point scale factor, k , is required to compute map projection distortion for a point on the ground. Because some surveying, engineering, and GIS software does not provide k , formulas for computing it are given below for the Transverse Mercator and Lambert Conformal Conic projections. These equations were derived from those provided in *NOAA Manual NOS NGS 5 "State Plane Coordinate System of 1983"* by James Stem (1990). Equations for computing the convergence angle of these projections are also provided.

For the Transverse Mercator projection, the grid scale factor at a point can be computed as follows (modified from Stem, 1990, pp. 32-35):

$$k = k_0 \left\{ 1 + \frac{(\Delta\lambda \cos \varphi)^2}{2} \left(1 + \frac{e^2 \cos^2 \varphi}{1 - e^2} \right) \left[1 + \frac{(\Delta\lambda \cos \varphi)^2}{12} \left(5 - 4 \tan^2 \varphi + \frac{e^2 \cos^2 \varphi}{1 - e^2} (9 - 24 \tan^2 \varphi) \right) \right] \right\}$$

where $\Delta\lambda = \lambda_0 - \lambda$ (in radians, for negative west longitude)

λ_0 = central meridian longitude

λ = geodetic longitude of point

φ = geodetic latitude of point

e^2 = reference ellipsoid first eccentricity squared

The following shorter equation can be used to approximate k for the Transverse Mercator projection. It is accurate to better than 0.02 part per million (at least 7 decimal places) if the computation point is within about $\pm 1^\circ$ of the central meridian (about 50 to 60 miles between latitudes of 30° and 45°):

$$k \approx k_0 \left\{ 1 + \frac{(\Delta\lambda \cos \varphi)^2}{2} \left(1 + \frac{e^2 \cos^2 \varphi}{1 - e^2} \right) \right\}$$

Note that this equation may not be sufficiently accurate for computing k throughout a UTM system zone (at the zone width of $\pm 3^\circ$ from the central meridian the error can exceed 1 ppm).

An even simpler equation can be used to approximate the grid scale factor, which utilizes the grid coordinate easting value and is about twice as accurate as the previous equation (i.e., better than 0.01 part per million if the computation point is within about $\pm 1^\circ$ of the central meridian):

$$k \approx k_0 + \frac{(E_0 - E)^2}{2(k_0 R)^2}$$

where E = Easting of the point where k is computed (in same units as R_G)

E_0 = False easting (on central meridian) of projection definition (in same units as R_G)

R = Earth geometric mean radius of curvature

For the Lambert Conformal Conic projection, the grid scale factor at a point can be computed as follows (modified from Stem, 1990, pp. 26-29):

$$k = k_0 \frac{\cos \varphi_C}{\cos \varphi} \sqrt{\frac{1 - e^2 \sin^2 \varphi}{1 - e^2 \sin^2 \varphi_C}} \exp \left\{ \frac{\sin \varphi_C}{2} \left[\ln \frac{1 + \sin \varphi_C}{1 - \sin \varphi_C} - \ln \frac{1 + \sin \varphi}{1 - \sin \varphi} + e \left(\ln \frac{1 + e \sin \varphi}{1 - e \sin \varphi} - \ln \frac{1 + e \sin \varphi_C}{1 - e \sin \varphi_C} \right) \right] \right\}$$

where k_0 = projection grid scale factor applied to central parallel (tangent to ellipsoid if $k_0 = 1$)

φ_C = geodetic latitude of central parallel = standard parallel for one-parallel LCC

$e = \sqrt{e^2} = \sqrt{2f - f^2}$ = first eccentricity of the reference ellipsoid

and all other variables are as defined previously. To use this equation for a two-parallel LCC, the two-parallel LCC must first be converted to an equivalent one-parallel LCC by computing φ_C and k_0 . The equations to do this are long, but are provided here for the sake of completeness. For a two-parallel LCC, the central parallel is

$$\varphi_C = \sin^{-1} \left[\frac{2 \ln \left(\frac{\cos \varphi_S}{\cos \varphi_N} \sqrt{\frac{1 - e^2 \sin^2 \varphi_N}{1 - e^2 \sin^2 \varphi_S}} \right)}{\ln \left(\frac{1 + \sin \varphi_N}{1 - \sin \varphi_N} \right) - \ln \left(\frac{1 + \sin \varphi_S}{1 - \sin \varphi_S} \right) + e \left[\ln \left(\frac{1 + e \sin \varphi_S}{1 - e \sin \varphi_S} \right) - \ln \left(\frac{1 + e \sin \varphi_N}{1 - e \sin \varphi_N} \right) \right]} \right],$$

and the central parallel scale factor is

$$k_0 = \frac{\cos \varphi_N}{\cos \varphi_C} \sqrt{\frac{1 - e^2 \sin^2 \varphi_0}{1 - e^2 \sin^2 \varphi_N}} \times \exp \left\{ \frac{\sin \varphi_C}{2} \left[\ln \left(\frac{1 + \sin \varphi_N}{1 - \sin \varphi_N} \right) - \ln \left(\frac{1 + \sin \varphi_C}{1 - \sin \varphi_C} \right) + e \left(\ln \left[\frac{1 + e \sin \varphi_C}{1 - e \sin \varphi_C} \right] - \ln \left[\frac{1 + e \sin \varphi_N}{1 - e \sin \varphi_N} \right] \right) \right] \right\},$$

where φ_N and φ_S = geodetic latitude of northern and southern standard parallels, respectively, and all other variables are as defined previously.

Convergence angles. For the TM, the convergence angle can be approximated as $\gamma = -\Delta\lambda \sin \varphi$ (where all variables are as defined previously; the units of γ are the same as the units of $\Delta\lambda$). This equation is accurate to better than $\pm 00.2''$ if the computation point is within $\sim 1^\circ$ of the central meridian. For any LCC, the convergence angle is exactly $\gamma = -\Delta\lambda \sin \varphi_C$.